Econ 5110 Solutions to the Midterm Exam

Spring 2008

1. Ramsey Growth Model (50 pts). Consider a Ramsey economy where agents have log utility, technology grows at rate g, and labor supply is fixed. The following system of differential equations describes the dynamics of this Ramsey economy

$$\begin{aligned}\frac{\dot{c}(t)}{c(t)} &= f_k(k(t)) - \rho - g \\ \dot{k}(t) &= f(k(t)) - c(t) - gk(t),\end{aligned}$$

where $f(k(t)) = k(t)^{\alpha}$, $k(0)$ is given, and all other notation is the same as in class.

(a) (12.5 pts) Find the steady-state level of capital, k^*.

Solution. The steady-state level is

$$k^* = \left(\frac{\rho + g}{\alpha}\right)^{1/(\alpha-1)}$$

(b) (12.5 pts) Draw the phase diagram for the dynamics of c and k. Make sure to carefully label the axes, phase arrows and saddle path.

Solution. See Figure 2.3 in Romer, 3rd edition.

(c) (12.5 pts) Find the golden-rule level of capital, k_g, and provide some intuition for the difference between k^* and k_g.

Solution. The golden-rule level is

$$k_g = \left(\frac{g}{\alpha}\right)^{1/(\alpha-1)}.$$

(d) (12.5 pts) Find the effects of a small change in the discount rate, ρ, on the steady-state savings rate, $s^* = [f(k^*) - c^*]/f(k^*)$. Does the sign on this derivative make intuitive sense? Explain.

Solution. The steady-state savings rate can be written as

$$s^* = \frac{g\alpha}{\rho + g}.$$

The derivative with respect to the discount rate is

$$\frac{\partial s^*}{\partial \rho} = -\frac{g\alpha}{(\rho + g)^2} \leq 0.$$

Yes, this makes intuitive sense. A higher discount rate leads to more current consumption and a smaller fraction of output being saved.

(e) **BONUS** (10 pts) Consider the effects of an increase in government spending (i.e., "the surge") for the war in Iraq. Modify the differential equations above and show in the phase diagram how this is predicted to impact the economy.

Solution. The introduction of government spending, $G(t)$, will change the capital accumulation equation to

$$\dot{k}(t) = f(k(t)) - c(t) - G(t) - gk(t),$$

and cause a shift down in the $\dot{k} = 0$ locus. This will not effect k^*, but it will reduce the steady-state consumption level, c^*.

2. Real Business Cycle Theory (50 pts). Consider a neoclassical growth model (notation similar to class) where all agents are identical and a representative agent maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \{g(c_t) + \phi \ln(l_t)\}$$

by choosing $\{c_t, l_t\}_{t=0}^{\infty}$ subject to

$$c_t + k_{t+1} \leq (1-\delta)k_t + y_t$$

where $g(c_t) = \ln(c_t)$, $y_t = s_t k_t^{1-\alpha} n_t^{\alpha}$, $l_t = 1 - n_t$, $s_t = s_{t-1}^{\rho} \exp(\epsilon_t)$, $\epsilon_t \sim iid(0, \sigma^2)$ and (s_0, k_0) given.

(a) (12.5 pts) Calculate the Euler equation for consumption, c_t. Provide some economic intuition for the equation.

Solution. The Euler equation is

$$c_t^{-1} = \beta E_t \left[c_{t+1}^{-1} \left((1-\delta) + (1-\alpha) \frac{y_{t+1}}{k_{t+1}} \right) \right].$$

This equation says that along the optimal path, the marginal utility of consumption at time t must equal the discounted, expected marginal utility of foregone consumption at time $t+1$.

(b) (12.5 pts) Describe the time paths of k, c, y and n to a one-time shock to ϵ_t when $\rho = 0.95$. Repeat with $\rho = 0$. Highlight the differences using a pair of stylized impulse response functions.

Solution. When $\rho = 0.95$, labor hours and output increase initially. Consumption also increases but not as much labor hours or output. Over time, labor hours and output gradually return to their steady states as the technology shock dies off. Consumption is smoother and declines much slower than labor hours and output. Capital stock dynamics are humped-shaped as agents initially save to finance their increased consumption.

When $\rho = 0$, the technology shock increases in the initial period and then immediately returns to its steady state. Labor hours and and output increase more than with the persistent shock. Consumption is smooth but increases much less due to a smaller increase in permanent income. Capital is also hump-shaped but increases less than with the persistent technology shock.

(c) (12.5 pts) The standard RBC model fails to produce sufficient variation in total hours worked to match the historical U.S. data. Describe three modifications to the basic model that will bring theory closer to the data.

Solution. Three modifications are (i) distributed lag of leisure, (ii) varying workweek of capital, and (iii) allowing for variation in hours along the extensive margin.

(d) (12.5 pts) Calculate the Euler equation for consumption when $g(c_t) = c_t - \theta c_t^2$. Are there any necessary restrictions on θ to make this a well-defined problem? Explain.

Solution. The Euler equation is

$$(1 - 2\theta c_t) = \beta E_t \left[(1 - 2\theta c_{t+1}) \left((1 - \delta) + (1 - \alpha) \frac{y_{t+1}}{k_{t+1}} \right) \right].$$

The necessary restriction on θ ensures that the marginal utility of consumption is positive and decreasing in c. This amounts to the restriction $0 < \theta \leq 0.5c^{-1}$.

(e) **BONUS** (10 pts) Imagine that manufacturing output contributed to the stock of an airborne pollutant that negatively impacted the well-being of households. How would you incorporate this feature into the RBC model? Discuss the equilibrium outcome of a decentralized economy versus that of a benevolent social planner.

Solution. We could let the aggregate stock of the pollutant, E, enter the utility function in a negative manner: $u = u(c, l, E)$ with $u_E < 0$. The law of motion for the pollutant is

$$E_{t+1} = (1 - \delta_E) E_t + h(Y_t),$$

where $h(Y_t)$ is an increasing function in aggregate output. Assuming E enters the utility function in a separable manner, then the decentralized equilibrium is unchanged by the pollutant except that the welfare of agents is reduced. The social planner, however, would act to internalize the pollution externality. For example, she could place a per-unit tax on the production of the manufacturing good.