ECON 5110 Solution to the Final Exam
Spring 2010

1. **Modified Cobweb Model.** (60 pts) Let the demand for a good \( d \) be given by

\[
d_t = \alpha_0 - \alpha_1 p_t + \alpha_2 y_t + \alpha_3 E_t y_{t+1} + \epsilon_t^d; \quad \text{(Demand)}
\]

supply \( s \) be given by

\[
s_t = \beta_0 + \beta_1 E_t - 2 p_t + \epsilon_t^s, \quad \text{(Supply)}
\]

where \( p \) is price; income \( y \) is exogenous and follows a mean-zero first-order autoregressive process with persistence parameter \( \lambda_y \); \( E_t - j \) is the expectations operator conditional on time \( t - j \) information; \( \epsilon_t^d \) and \( \epsilon_t^s \) are mean-zero, mutually independent white-noise error terms; all \( \alpha \)'s and \( \beta \)'s are positive; and the market clears (i.e., \( d_t = s_t \), for all \( t \)).

(a) (10 pts) Provide an economic interpretation for the two modifications to the basic cobweb model.

Solution. The first modification is the addition of \( y_t \) and \( E_t y_{t+1} \) to the demand function. The economic interpretation is that current and expected future income increase permanent income and the demand for the good. The second modification is that supply depends on the two-period (as opposed to one-period) ahead forecast of price. The economic interpretation is that the good has a two-period gestation period before it is available for the market.

(b) (10 pts) Find the steady state.

Solution. The steady state is given by

\[
p = \frac{\alpha_0 - \beta_0}{\alpha_1 + \beta_1} \quad \text{and} \quad d = s = \frac{\beta_0 \alpha_1 + \beta_1 \alpha_0}{\alpha_1 + \beta_1}.
\]

(c) (10 pts) Find the equilibrium under naïve expectations. Describe the transition dynamics in words and using a diagram.

Solution. The equilibrium under naïve expectations is

\[
p_t = \alpha_1^{-1} \left[ (\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3) y_t - \beta_1 p_{t-2} + (\epsilon_t^d - \epsilon_t^s) \right],
\]

for \( t = 2, \ldots, T \) given \( p_0 \). Assuming \( \alpha_1 > \beta_1 \), the equilibrium involves a two-period price cycle that converges to the steady state, \( p \).

(d) (10 pts) Find the fundamental rational expectations equilibrium (REE).

Solution. The REE is

\[
p_t = p + \delta y_{t-2} + \eta_t,
\]
where

\[ \delta = \frac{(\alpha_2 + \alpha_3\lambda_y)\lambda_y^2}{(\alpha_1 + \beta_1)} \quad \text{and} \quad \eta_t = \alpha_1^{-1}[(\epsilon_t^d - \epsilon_t^s) + (\alpha_2 + \alpha_3\lambda_y)(\lambda_y\epsilon_t^t - \epsilon_t^s)]. \]

(c) (10 pts) Is a rational bubble possible in equilibrium?

**Solution.** A rational bubble equilibrium must satisfy

\[(p_t + B_t) = \alpha_1^{-1}[(\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3\lambda_y)y_t - \beta_1 E_{t-2}(p_t + B_t) + (\epsilon_t^d - \epsilon_t^s)].\]

which implies that

\[ B_t = -\frac{\beta_1}{\alpha_1} E_{t-2}B_t. \]

Taking expectations conditional on time \(t - 2\) information implies that

\[ E_{t-2}B_t = -\frac{\beta_1}{\alpha_1} E_{t-2}B_t \implies B_t = E_{t-2}B_t = 0. \]

Therefore a rational bubble is not possible.

(f) (10 pts) Under what conditions is the REE stable under least squares learning?

**Solution.** The perceived law of motion (PLM) is

\[ p_t = \pi_0 + \pi_1 y_{t-2} + v_t. \]

The actual law of motion (ALM) is

\[
\begin{align*}
p_t &= \alpha_1^{-1}[(\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3\lambda_y)y_t - \beta_1 E_{t-2}p_t + (\epsilon_t^d - \epsilon_t^s)] \\
&= \alpha_1^{-1}[(\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3\lambda_y)y_t - \beta_1(\pi_0 + \pi_1 y_{t-2}) + (\epsilon_t^d - \epsilon_t^s)] \\
&= \alpha_1^{-1}[(\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3\lambda_y)\lambda_y^2 y_{t-2} - \beta_1(\pi_0 + \pi_1 y_{t-2})] + \eta_t \\
&= \tilde{\pi}_0 + \tilde{\pi}_1 y_{t-2} + \eta_t.
\end{align*}
\]

The REE is stable under learning if the following differential equations are locally, asymptotically stable:

\[
\begin{align*}
\frac{d\pi_0}{dt} &= \tilde{\pi}_0 - \pi_0 = \alpha_1^{-1}[(\alpha_0 - \beta_0) - \beta_1 \pi_0] - \pi_0 \quad \text{and} \\
\frac{d\pi_1}{dt} &= \tilde{\pi}_1 - \pi_1 = \alpha_1^{-1}[(\alpha_2 + \alpha_3\lambda_y)\lambda_y^2 - \beta_1 \pi_1] - \pi_1
\end{align*}
\]

This condition is satisfied if \(-(1 + \beta_1/\alpha_1) < 0.\) The REE is stable under learning because \(\beta_1 > 0\) and \(\alpha_1 > 0.\)
2. Dynamic New Keynesian (DNK) Model. (40 pts) Consider the following DNK model,

\[
\begin{align*}
    x_t &= -\varphi[i_t - E_t \pi_{t+1}] + \kappa E_t x_{t+1} + \epsilon_t \quad \text{(IS curve)} \\
    \pi_t &= \lambda x_t + \gamma E_t \pi_{t+1} + \mu_t \quad \text{(Phillips curve)} \\
    i_t &= i + \theta_x E_t x_{t+1} + \theta_{E_t} E_t \pi_{t+1} \quad \text{(Taylor rule)}
\end{align*}
\]

where the variable definitions are the same as those discussed in class.

(a) (10 pts) Discuss the relationship between the intertemporal elasticity of substitution for consumption and the slope of the IS curve.

Solution. The intertemporal elasticity of substitution (IES) measures the willingness of individuals to allocate consumption across time in response to changes in rates of return. As a result, an economy comprised of individuals with a high IES will have a large response to changes in the real interest rate, which implies a flat IS curve. Conversely, an economy comprised of individuals with a low IES will lead to a steep IS curve.

(b) (10 pts) Solve for the REE under the special case where \( \kappa = \theta_x = 0 \). Use the restriction for parts (c) and (d).

Solution. The REE is

\[
\pi_t = \lambda \left[ -\varphi [i + \theta_x E_t x_{t+1} - E_t \pi_{t+1}] + \epsilon_t \right] + \gamma E_t \pi_{t+1} + \mu_t
\]

where \( a = \lambda \varphi i \) and \( b = \varphi \lambda (\theta_x - 1) + \gamma \). If we assume that \( |b| < 1 \), then the reduced-form REE is given by

\[
\pi_t = \frac{a}{1 - b} + v_t.
\]

(c) (10 pts) Discuss the Taylor principle and how it relates to determinacy of the REE. How aggressive does the central bank need to be against inflation to avoid an indeterminate equilibrium?

Solution. The Taylor principle states that the central banks must change nominal interest rates more than one-for-one with changes in expected inflation (i.e., \( \theta_{\pi} > 1 \)) to avoid an indeterminate REE. For the model above with \( \kappa = \theta_x = 0 \) and assuming \( b > 0 \), the REE is regular if

\[
b = -\varphi \lambda (\theta_{\pi} - 1) + \gamma < 1
\]

or

\[
\theta_{\pi} > 1 + \frac{\gamma - 1}{\varphi \lambda}.
\]
(d) (10 pts) Calculate the condition under which the liquidity trap will bind in the REE.

Solution. The liquidity trap will bind when \( i_t \leq 0 \) from the interest-rate rule. In the REE, this will occur when

\[
i_t = i + \theta \pi \frac{a}{1 - b} = i + \frac{-\lambda \varphi \pi \theta}{1 + \varphi \lambda (\theta - 1) - \gamma} = \frac{(1 - \varphi \lambda - \gamma)i}{1 + \varphi \lambda (\theta - 1) - \gamma} \leq 0.
\]