

# ECON 5110 Solutions to the Final Exam

Spring 2008

1. **Money Demand, Expectations and Learning.** (50 pts) Consider the simple money demand function

$$\frac{M_t}{P_t} = \left( \frac{E_t P_{t+1}}{P_t} \right)^\alpha \quad (1)$$

where the money supply follows  $M_t = M \exp(\epsilon_t)$ ,  $M$  is a constant and  $\epsilon_t$  is mean-zero white noise.

- (a) (10 pts) Linearize the system and write equation (1) in our standard forward-looking representation.

Denote percentage deviations from steady state in lower case.

Answer. The linearized version of equation (1) is

$$m_t - p_t = \sigma(E_t p_{t+1} - p_t)$$

and the linearized version of the money supply process is

$$m_t = m + \epsilon_t.$$

The standard forward-looking form is

$$p_t = bE_t p_{t+1} + cm_t$$

where  $b = -\alpha/(1 - \alpha)$  and  $c = 1/(1 - \alpha)$ .

- (b) (10 pts) Find the equilibrium under naive expectations. Does this equilibrium make intuitive sense?

Explain.

Answer. The equilibrium under naive expectations is

$$p_t = m + \epsilon_t \Rightarrow p_t - m_t = 0.$$

This makes intuitive sense because under naive expectations, agents expect no inflation and thus the demand for real money balances will remain constant.

- (c) (10 pts) Find the rational expectations equilibrium (REE).

Answer. The rational expectations equilibrium (REE) is

$$p_t = \frac{cm}{1 - b} + c\epsilon_t$$

provided  $b$  is less than one in absolute value. If  $b$  is greater than one in absolute value, the solution is

$$p_{t+1} = (1/b)p_t - (c/b)m_t + \eta_{t+1}$$

where  $\eta_{t+1}$  is the rational expectations forecast error.

- (d) (10 pts) What parameter values would lead to an indeterminate equilibrium? What would this imply about the demand for money?

Answer. If  $\alpha > 0.5$ , then  $b$  would be greater than one in absolute value and the REE would be indeterminate. This would imply that the demand for real money balances would be increasing in inflation, which is counter-intuitive.

- (e) (10 pts) Under what conditions is the REE stable under least squares learning?

Answer. The perceived law of motion for  $p_t$  is

$$p_t = \phi + v_t.$$

This implies that the actual law of motion for  $p_t$  is

$$p_t = b\phi + cm + c\epsilon_t.$$

If the REE is locally asymptotically stable under the differential equation

$$\frac{d\phi}{d\tau} = \phi(b - 1) + cm,$$

then the REE is said to be stable under least-squares learning. This is true if  $b$  is less than one, which is guaranteed when money demand depends negatively on expected inflation.

2. **Dynamic New Keynesian (DNK) Model.** (30 pts) Consider the following DNK model,

$$x_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + \epsilon_t \quad (\text{IS curve})$$

$$\pi_t = \lambda x_t + \gamma E_t \pi_{t+1} + \mu_t \quad (\text{Phillips curve})$$

$$i_t = i + \theta_x E_t x_{t+1} + \theta_\pi E_t \pi_{t+1} \quad (\text{Taylor rule})$$

where the variable definitions are the same as those discussed in class.

- (a) (10 pts) Let the household objective function be

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} (1 - \sigma)^{-1} c_{t+i}^{1-\sigma}.$$

Briefly discuss the behavior of  $\varphi$  as  $\sigma \rightarrow \infty$ . What does this imply for the slope of the IS curve? Will monetary policy be more or less effective in stabilizing output?

Answer. As  $\sigma \rightarrow \infty$ , households become very reluctant to substitute consumption across time periods in response to variations in the rate of return. As a result, output ( $x_t$ ) is not sensitive to changes in the

real interest rate, i.e.,  $\varphi$  goes to zero. This implies that the IS curve is vertical and monetary policy will be ineffective in stabilizing output.

- (b) (10 pts) Assume the Phillips curve is derived from a Calvo-style sticky price model where the probability of changing price in any period is given by  $(1 - \nu)$ . Briefly discuss the behavior of  $\lambda$  as  $\nu \rightarrow 0$ . What does this imply for the slope of the Phillips curve and the macro effects of demand-side shocks  $\epsilon_t$ ?

Answer. As  $\nu \rightarrow 0$ , the DNK model approaches the flexible-price Classical model and the Phillips curve will be vertical even in the short run. Therefore, we expect  $\lambda \rightarrow \infty$ . This implies that demand-side shocks will not affect output and will only lead to variation in inflation.

- (c) (10 pts) Discuss the Taylor principle and how it relates to the magnitude of  $\theta_\pi$ . Provide some economic intuition for the stabilizing effects of a central bank that follows the Taylor principle.

Answer. The Taylor principle states that in order for policymakers to effectively fight inflation, the nominal interest rate must rise more than expected inflation so that the real interest rate increases (i.e.,  $\theta_\pi > 1$ ). When the real interest rate increases this reduces aggregate demand and inflationary pressures, which tends to stabilize the macroeconomy.

3. **Miscellaneous Questions.** (20 pts) Answer TWO of the four following questions.

- (a) (10 pts) Why are overlapping wage contracts an important mechanism for explaining the business cycle? In particular, how are the dynamics of the model different from the case where wage contracts do not overlap.
- (b) (10 pts) Describe in words why optimal monetary policy is time inconsistent and why discretion may lead to suboptimal outcomes.
- (c) (10 pts) What are the fundamental differences between the sticky-price and sticky-information models of the business cycle?
- (d) (10 pts) What is the primary cause of the Great Depression? What is the propagation mechanism in the economy that lead to the dramatic fall in output and the slow recovery.