Econ 5110 Midterm Exam

Spring 2009

Ramsey Model (100 pts). Consider a discrete-time version of the Ramsey model with a large number of identical agents who inelastically supply their labor (i.e., $l_t = 1$). The representative agent maximizes

$$E_0^* \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

by choosing $\{c_t\}_{t=0}^{\infty}$ subject to

$$k_{t+1} = y_t - c_t$$

where $y_t = s_t A_t k_t^{\alpha}$, $\alpha < 1$, and there is full depreciation. Technology shocks are random with zero persistence $s_t = \exp(\epsilon_t)$, $\epsilon_t \sim iid(0, \sigma^2)$. The A_t term represents an economy-wide production externality ($A_t = k_t^{\theta}$) that individuals take as given when making consumption decisions. Initial capital k_0 is given. Begin by assuming that $\theta = 0$.

1. (20 pts) Calculate the Euler equation for consumption, c_t. Provide some economic intuition for the equation.

2. (20 pts) Find the steady-state equilibrium for (c, y, k) in terms of the fundamental parameters. Differentiate the steady-state expressions for (c, y, k) with respect to β and determine the signs of the derivatives. Do the signs make sense? Explain.

3. (20 pts) Linearize the Euler equation (in terms of c_t, c_{t+1} and k_{t+1}), the production function (in terms of y_t, ϵ_t and k_t) and the law of motion for capital (in terms of k_{t+1}, y_t and c_t). Collapse the system to one second-order, stochastic linear difference equation in k and ϵ.

4. (20 pts) Find the equilibrium under naive expectations and describe the transition dynamics to a single positive technology shock.

5. (20 pts) Find the rational expectations equilibrium (REE) in the presence of a production externality ($\alpha + \theta > 1$) and discuss the nature of the equilibrium.

   Bonus. (10 pts). The rational expectations equilibrium (REE) is of the form: $\hat{k}_{t+1} = \phi_1 \hat{k}_t + \phi_2 \epsilon_t$. Use undetermined coefficients to solve for ϕ_1 and ϕ_2 in terms of the fundamental parameters.