1. Re-work the simple endogenous growth model in Section 2 of our class notes with \( 0 < \delta < 1 \).

**Answer two of the following three questions.**

2. Consider the production function \( Y = AK + BL \), where \( A \) and \( B \) are positive constants. Also assume constant population growth at rate \( n \) and constant depreciation at rate \( \delta \). [Barro and Sala-i-Martin, 2004]

(a) Write output per person as a function of capital per person. What is the marginal product of \( k \)?
   What is the average product of \( k \)?

(b) Write down the fundamental dynamic equation for the Solow model.

(c) Under what conditions does this model have a steady state with no growth of \( k \), and under what conditions does the model display endogenous growth?

(d) In the case of endogenous growth, how does the growth rate of the capital stock behave over time (that is, does it increase or decrease)? What about the growth rates of output and consumption per capita?

(e) If \( s = 0.4 \), \( A = 1 \), \( B = 2 \), \( \delta = 0.08 \), and \( n = 0.02 \), what is the long-run growth rate of this economy? What if \( B = 5 \)? Explain the differences.

3. This problem is illustrates the crucial role played by the assumption of constant returns in new growth theories. The framework is one of a simple neoclassical model with a constant saving rate. The production function for firm \( i \) is

\[
y_i(t) = A(t)\eta k_i(t)^\alpha
\]

where \( 0 < \alpha < 1 \), \( A(t) = \frac{1}{N} \sum_{i=1}^{N} k_i(t) \), and \( N \) is the number of firms. Suppose that \( s \) is the constant saving rate, \( n \) is the constant population growth rate, and \( \delta \) is the rate of depreciation of physical capital.

(a) Find the differential equation for \( k \) when all firms are identical.

(b) Represent graphically the solutions to the model for the cases where the production function exhibits (i) diminishing returns to scale, \( \alpha + \eta < 1 \), (ii) constant returns to scale, \( \alpha + \eta = 1 \), and (iii) increasing returns to scale, \( \alpha + \eta > 1 \). What is meant by the "knife-edge" property of the AK model? Explain.

(c) Examine the effect on the long-run growth rate of a change in the saving rate for each of the three cases.
(d) Consider the effect of a once-off shock. Suppose an earthquake destroys half the capital stock of the economy. Examine what happens in each of the three cases, both in the short run and long run.

4. Rework the Green Solow model assuming AK growth. Specifically, how does this effect Figure 4 and the test for conditional emission convergence?