

## 1 Introduction

In this section, I present the details of a basic real business cycle (RBC) model. I rely heavily on Prescott's (1986) seminal paper and the material in Farmer. Chapter 4 of Romer also provides a nice summary.

## 2 Real Business Cycle Theory

### 2.1 Prescott (1986) "Theory Ahead of Business Cycle Measurement"

Prescott presents the following puzzle: Why do "...industrial market economies display recurrent, large fluctuations in output and employment over relatively short time periods....when the associated movements in labor's marginal product are small"? The surprising answer, according to Prescott, is that there is no puzzle – this is exactly "what standard economic theory predicts."

RBC theory explains business cycles by incorporating stochastic technology shocks into a standard Neo-classical growth (Ramsey) model. Agents in the economy vary (both intra- and inter-temporally) their consumption and leisure choices in response to technology shocks that alter the returns to working. This endogenous variation in consumption and leisure creates business cycles.

### 2.2 The Model

Begin by assuming that a representative agent maximizes her discounted future utility, which is given by

$$J = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [(1 - \phi) \log(c_t) + \phi \log(1 - n_t)] \right\}$$

by choosing  $\{c_t, n_t\}_{t=0}^{\infty}$  subject to the following constraints

- $x_t + c_t \leq s_t k_t^{1-\theta} n_t^\theta$  (resource constraint)
- $k_{t+1} = (1 - \delta)k_t + x_t$  (capital accumulation)
- $s_{t+1} = s_t^\rho \exp(\epsilon_{t+1})$  (technology shock process)
- $\epsilon_{t+1} \sim iid(0, \sigma^2)$  (driving shock)
- $k_0$  and  $s_0$  given.

### 2.2.1 First-Order Conditions

To see the first-order conditions, write out the objective function starting at period  $t$  with the constraints substituted in

$$J = (1 - \phi) \log[-(k_{t+1} - (1 - \delta)k_t) + s_t k_t^{1-\theta} n_t^\theta] + \phi \log(1 - n_t) + \\ E_t \beta \{ (1 - \phi) \log[-(k_{t+2} - (1 - \delta)k_{t+1}) + s_{t+1} k_{t+1}^{1-\theta} n_{t+1}^\theta] + \phi \log(1 - n_{t+1}) \} + \dots$$

The first-order conditions are found by taking the derivatives with respect to  $k_{t+1}$  (or equivalently  $c_t$ ) and  $n_t$  and setting them equal to zero. Begin with capital:

$$\frac{\partial J}{\partial k_{t+1}} = -(1 - \phi) \frac{1}{c_t} + E_t [\beta (1 - \phi) \frac{1}{c_{t+1}} ((1 - \delta) + (1 - \theta) s_{t+1} k_{t+1}^{-\theta} n_{t+1}^\theta)] = 0 \quad (1)$$

which after rearranging gives

$$\frac{(1 - \phi)}{c_t} = \beta E_t \left[ \frac{(1 - \phi)}{c_{t+1}} \left( (1 - \delta) + (1 - \theta) \frac{y_{t+1}}{k_{t+1}} \right) \right]. \quad (2)$$

The intuition behind (2) is straightforward. To maximize future utility, the agent must equate the utility of an extra unit of consumption today ( $\frac{\partial U_t}{\partial c_t}$ ) and the discounted, expected utility of foregone consumption tomorrow ( $\frac{\partial U_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial y_{t+1}} \frac{\partial y_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial x_t}$ ) for each period  $t = 0, \dots, \infty$ . The first-order condition with respect to labor is

$$\frac{\partial J}{\partial n_t} = (1 - \phi) \frac{1}{c_t} \theta s_t k_t^{1-\theta} n_t^{\theta-1} + \phi \frac{-1}{1 - n_t} = 0 \quad (3)$$

which after rearranging gives

$$\frac{(1 - \phi)}{c_t} \theta \frac{y_t}{n_t} = \frac{\phi}{1 - n_t}. \quad (4)$$

The intuition behind (4) is that the extra utility from consuming the fruits of labor ( $\frac{\partial U_t}{\partial c_t} \frac{\partial c_t}{\partial y_t} \frac{\partial y_t}{\partial n_t}$ ) must equal to the disutility of labor ( $-\frac{\partial U_t}{\partial n_t}$ ). Finally, combining the resource constraint and capital accumulation equation gives

$$k_{t+1} = (1 - \delta)k_t - c_t + s_t k_t^{1-\theta} n_t^\theta. \quad (5)$$

The equilibrium is therefore a nonlinear (expectational) system of four difference equations – equations (2), (4), (5) and the law of motion for  $s_t$  – in four variables ( $s_t, k_t, c_t, n_t$ ).

### 2.3 Solving the System

There are many different solution methods for nonlinear, rational-expectations difference equations such as those above. A good reference is the book *Frontiers of Business Cycle Research*, which is a collection of

papers edited by Thomas Cooley. Below I present the method outlined by Farmer, which is a variation of the procedure of Blanchard and Kahn (1980) in *Econometrica*.

### 2.3.1 Steady State

The steady state is defined as the values all variables would converge to in the absence of any technology shocks. The steady-state value of a variable is denoted by the lack of a time subscript. The system of steady-state equations are

$$1 = \beta \left( (1 - \delta) + (1 - \theta) \frac{y}{k} \right) \quad (6)$$

$$\frac{\phi}{1 - n} = \frac{(1 - \phi) \theta y}{c n} \quad (7)$$

$$y = k^{1 - \theta} n^\theta \quad (8)$$

$$k = (1 - \delta)k - c + k^{1 - \theta} n^\theta. \quad (9)$$

Given values for the structural parameters  $(\beta, \delta, \theta, \phi)$ , the system can be solved for the steady-state values  $y, k, c$  and  $n$ .

### 2.3.2 Linearizing the Transition Dynamics

For pedagogical purposes, assume  $n_t = 1$  for all periods so that equations (4) and (7) are not relevant. The first problem in solving for the transitional dynamics is that equations (2), (5) and the law of motion for technology shocks are nonlinear. Standard solution techniques for rational expectations models require a linear system. Consider taking a first-order Taylor series expansion to the system around the steady state. For an arbitrary equation  $x_t = f(y_t, z_t)$ , the linearization (with variables written in proportional deviations from their steady states) will take the form

$$\begin{aligned} \frac{x_t - x}{x} &= f_y(y) \frac{y}{x} \frac{y_t - y}{y} + f_z(z) \frac{z}{x} \frac{z_t - z}{z} + \eta_t \Rightarrow \\ \hat{x}_t &= a_y \hat{y}_t + a_z \hat{z}_t + \eta_t \end{aligned}$$

where  $\eta_t$  is an approximation error and hats over variables denote proportional deviations from the steady state. The linearized versions of (2), (5) and the law of motion for technology shocks are

$$\hat{c}_t = E_t \hat{c}_{t+1} + a_2 E_t \hat{k}_{t+1} + a_3 E_t \hat{s}_{t+1} \quad (10)$$

$$\hat{k}_{t+1} = a_4 \hat{k}_t + a_5 \hat{s}_t + a_6 \hat{c}_t \quad (11)$$

$$\hat{s}_{t+1} = \rho \hat{s}_t + \epsilon_{t+1} \quad (12)$$

where the coefficients are defined as

- $a_2 = \beta\theta(1 - \theta)k^{-\theta}$
- $a_3 = -\beta(1 - \theta)k^{-\theta}$
- $a_4 = (1 - \delta) + (1 - \theta)k^{-\theta}$
- $a_5 = k^{-\theta}$
- $a_6 = -c/k$ .

### 2.3.3 Standard Form

The next step is to write the system in standard matrix form, where we define the conditional expectation of  $\hat{c}_{t+1}$  to be  $E_t(\hat{c}_{t+1}) = \hat{c}_{t+1} + w_{t+1}^c$ , where  $w_{t+1}^c$  is an expectational error that satisfies  $E_t w_{t+1}^c = 0$ . Similarly for  $\hat{k}_{t+1}$  and  $\hat{s}_{t+1}$ . The system in matrix form is

$$\begin{bmatrix} 1 & 0 & 0 \\ a_6 & a_4 & a_5 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix} = \begin{bmatrix} 1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{s}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{t+1} \\ w_{t+1}^c \\ w_{t+1}^k \\ w_{t+1}^s \end{bmatrix}$$

or written more compactly as

$$Y_t = AY_{t+1} + BV_{t+1} \tag{13}$$

where  $Y_t = (\hat{c}_t, \hat{k}_t, \hat{s}_t)'$ ,  $V_{t+1} = (\epsilon_{t+1}, w_{t+1}^c, w_{t+1}^k, w_{t+1}^s)'$ ,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a_6 & a_4 & a_5 \\ 0 & 0 & \rho \end{bmatrix}^{-1} \begin{bmatrix} 1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 & 0 \\ a_6 & a_4 & a_5 \\ 0 & 0 & \rho \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

### 2.3.4 Diagonalization

The dynamic properties of the model depend critically on the matrix  $A$ . It will be convenient to calculate the roots or eigenvalues of the matrix  $A$ , (i.e., the  $\lambda$ s that solve the following matrix equation  $(A - \lambda I)Y = 0$ .) The interesting solution to this problem requires that  $(A - \lambda I)$  be singular, or in other words,  $|A - \lambda I| = 0$ .

Solving this equation produces the eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$  where  $n$  is the number of rows (or columns) of  $A$ . Eigenvalues less than one in absolute value (i.e., inside the unit circle) are called forward stable roots. The associated eigenvectors satisfy  $AY^{(i)} = \lambda_i Y^{(i)}$  for  $i = 1, \dots, n$ . For the RBC example above, stacking these equations together produces

$$A \begin{bmatrix} Y^{(1)} & Y^{(2)} & Y^{(3)} \end{bmatrix} = \begin{bmatrix} Y^{(1)} & Y^{(2)} & Y^{(3)} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

or  $AQ = Q\Lambda$  where  $Q$  is the matrix of stacked eigenvectors and  $\Lambda$  is the diagonal matrix with the eigenvalues along the main diagonal.

Diagonalization of  $A$  involves writing it in the form  $A = Q\Lambda Q^{-1}$ . We will use this representation of  $A$  to write the system as a set of independent equations.

### 2.3.5 Transformation

Begin by taking expectations of (13) conditional on time  $t$  information, which produces

$$Y_t = AE_t Y_{t+1}.$$

Using the diagonalization of  $A$  above, we get

$$Y_t = Q\Lambda Q^{-1}E_t Y_{t+1} \Rightarrow Q^{-1}Y_t = \Lambda E_t Q^{-1}Y_{t+1}.$$

If we let  $Z_t \equiv Q^{-1}Y_t$ , then we can write the matrix system as three independent equations

$$Z_t = \Lambda E_t Z_{t+1} \Rightarrow \begin{bmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \end{bmatrix} = \begin{bmatrix} \lambda_1 E_t z_{1,t+1} \\ \lambda_2 E_t z_{2,t+1} \\ \lambda_3 E_t z_{3,t+1} \end{bmatrix}. \quad (14)$$

### 2.3.6 Law of Iterated Expectations

Equations (14) hold for all  $t = 0, \dots, \infty$ . Using this fact, we can substitute for  $Z_t$  on the right-hand side of (14) to get

$$Z_t = \Lambda E_t [\Lambda E_{t+1} Z_{t+2}] = \Lambda^2 E_t Z_{t+2}$$

where we used the law of iterated expectations for the last equality. Repeated substitutions produce

$$Z_t = \Lambda^T E_t Z_{t+T}.$$

For the  $n_s$  forward-stable roots, if we let  $T \rightarrow \infty$  and impose the condition that the  $\lim_{T \rightarrow \infty} E_t Z_{t+T}$  does not explode too fast, then we have

$$z_{it} = 0$$

for  $i = 1, \dots, n_s$ .

### 2.3.7 Impose Constraint

The RBC model, written in this form, will possess one forward stable root (i.e.,  $n_s = 1$ ). Using this fact and the relationship  $Z_t \equiv Q^{-1}Y_t$ , we now have an additional restriction on the model

$$\hat{c}_t = q^{11}\hat{k}_t + q^{12}\hat{s}_t \tag{15}$$

where the  $q$  coefficients are associated with the first (i.e., forward-stable) row of  $Q^{-1}$ .

### 2.3.8 Vector Autoregression and the Policy Functions

Finally, we can substitute (15) into (11) to obtain

$$\begin{aligned} \hat{k}_{t+1} &= a_4\hat{k}_t + a_5\hat{s}_t + a_6(q^{11}\hat{k}_t + q^{12}\hat{s}_t) \\ &= (a_4 + a_6q^{11})\hat{k}_t + (a_5 + a_6q^{12})\hat{s}_t \\ &= b_1\hat{k}_t + b_2\hat{s}_t. \end{aligned}$$

Along with (12),  $k_0$  and  $s_0$ , we can describe the evolution of the state variables in vector autoregression (VAR) form as

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{s}_{t+1} \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{s}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_{t+1}$$

and the evolution of the remaining variables are determined through the following "policy functions"

$$(\hat{c}_t, \hat{y}_t, \hat{x}_t) = \Pi \begin{bmatrix} \hat{k}_t \\ \hat{s}_t \end{bmatrix}.$$

## 2.4 Calibration

Now back to Prescott. Rather than attempt to estimate the structural parameters via econometrics, Prescott calibrates the model by choosing parameter values that are consistent with long-run historical averages and microeconomic evidence. Prescott chooses the following values

- $\theta = 0.64$  (labor's share of national income)

- $\delta = 0.025$  (corresponding to about 10% depreciation per annum)
- $\phi = 2/3$  (productive time to non-market activities)
- $\rho = 1$  (random walk technology in logs)
- $\beta = 0.99$  (corresponds to a subjective discount rate of 4% per annum)
- $\sigma = 0.763$  (standard deviation of technology shocks from Solow residuals)

## 2.5 Simulation

Once the model has been calibrated and solved for its VAR form, it is possible to feed in counterfactual technology shocks (similar to those experienced in recent US history) and simulate artificial data on variables such as output, consumption, investment, and total hours worked. The properties of the artificial data can then be contrasted with the actual US data. Before discussing the results of such an exercise, we need to address the issue of trend-cycle decomposition.

### 2.5.1 Hodrick-Prescott Filter

In order to remove the long-run growth (trend) component from the U.S. data, Prescott employs a flexible detrending procedure known as the Hodrick-Prescott (HP) filter. The HP filter chooses the trend  $\tau_t$  (or equivalently the cycle  $Y_t - \tau_t$ ) to minimize

$$\sum_{t=1}^T (Y_t - \tau_t)^2 + \lambda \left\{ \sum_{t=2}^T [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\}$$

where  $\lambda$  is the smoothing parameter. Higher values of  $\lambda$  result in a smoother trend. For instance,

- $\lambda = 0$  results in  $\tau_t = Y_t$  and there are no cycles.
- $\lambda \rightarrow \infty$  results in  $\tau_t = a + bt$  or a linear trend.
- Prescott chooses  $\lambda = 1600$ , which causes the HP filter to focus on cycles with periodicity of 8 years or less.

### 2.5.2 Simulation Results for Postwar U.S. and RBC Economies

Prescott presents his findings for the HP-filtered U.S. economy between 1947 and 1982. The focus is on standard deviations of and comovements between various macro time series. Here is a sampling of the results reported in Figure 2 and Table 1:

#### U.S. Stylized Facts

- Hours worked and output are highly correlated over the business cycle.

- Hours worked and output are almost equally volatile.
- The standard deviation of consumption is less than the standard deviation of output.
- The standard deviation of investment is much greater than the standard deviation of output.
- The average product of labor  $Y/L$  is weakly procyclical.

Table 2 presents the average statistics for 20 simulations of the RBC economy:

#### RBC Facts

- The standard deviation of output is approximately 4/5ths of that in the U.S. economy.
- Consumption is less volatile than output.
- Investment is much more volatile than output.
- Hours worked are only about one-half as volatile as U.S. hours worked.
- The average product of labor is strongly procyclical.

The first three RBC facts led Prescott to claim that "given people's ability and willingness to intratemporally and intertemporally substitute consumption and leisure, ... it would be puzzling if the economy did not display these large fluctuations." The last two facts are failures for the model and have been a major source of further research among RBC theorists.

## 2.6 Modifications

Prescott discusses two possible modifications to the "plain vanilla" RBC model to increase the volatility of hours worked – adding a distributed lag of leisure and allowing for a varying workweek of capital.

### 2.6.1 Distributed Lag of Leisure

Kydland and Prescott modify the utility function as follows

$$u[c_t, \sum_{i=0}^{\infty} \alpha_i l_{t-i}] = \frac{1}{3} \log(c_t) + \frac{2}{3} \log(\sum_{i=0}^{\infty} \alpha_i l_{t-i})$$

where  $l_t = 1 - n_t$  is leisure,  $\sum_{i=0}^{\infty} \alpha_i = 1$ ,  $\alpha_0 = 0.5$ , and the remaining  $\alpha_i$  decline geometrically at rate 0.9. This preference structure captures the notion of "fatigue", that is, high work effort in the recent past increases the marginal utility of leisure today. To see this, note that

$$\frac{\partial u_t}{\partial l_t} = \frac{2}{3} \frac{0.5}{\sum_{i=0}^{\infty} \alpha_i l_{t-i}}.$$

In other words, low levels of leisure in the past increase the marginal utility of leisure today and hence the degree of intertemporal substitution of labor.

### 2.6.2 Varying Workweek for Capital

Allowing the workweek of capital to vary with the workweek of labor also increases the intertemporal substitution of labor. The typical chain of events in an RBC model is

$$\text{positive technology shock } s_t \Rightarrow \uparrow \frac{\partial y_t}{\partial n_t} \Rightarrow \uparrow n_t$$

so that hours worked increase in response to a technology shock that raises the returns to labor. With a varying utilization of capital, we get the additional path

$$\text{positive technology shock } s_t \Rightarrow \uparrow \frac{\partial y_t}{\partial n_t} \Rightarrow \uparrow n_t \Rightarrow \uparrow k_t \Rightarrow \uparrow \frac{\partial y_t}{\partial n_t} \Rightarrow \uparrow n_t$$

where higher capital utilization in response to more hours worked further increases the marginal product of labor and labor market activity.

### 2.6.3 Results

By adding these two modifications, the standard deviation of hours worked in the modified RBC economy is nearly 70% the standard deviation of output (as opposed to 50% in the basic RBC model). Furthermore, the standard deviation of output in the modified RBC economy is now approximately equal to that in the U.S. economy.

## 3 Economic Mechanisms

In this section, we look deeper into the economic mechanisms that generate business-cycle fluctuations. A useful starting point is to ask the following two questions

1. Why is consumption less and investment more volatile than output?
2. Why are hours worked so volatile over the business cycle?

Keynes would answer the first by saying firms have volatile expectations (animal spirits) and answer the second by saying there exists a failure of effective demand working through sticky wages. The neoclassical answers are much different and are treated in order below.

### 3.1 Consumption Smoothing

To see how consumption smoothing emerges in the neoclassical model, consider a simple two-period model where agents maximize lifetime utility

$$u(c_1, c_2) = \frac{c_1^{1-\eta}}{1-\eta} + \beta \frac{c_2^{1-\eta}}{1-\eta}$$

by choosing first-period consumption  $c_1$  and second-period consumption  $c_2$  subject to the budget constraint

$$c_2 = y_2 + (1 + R)(y_1 - c_1).$$

where  $R$  is the rate of return on savings. The Lagrangian function is

$$\mathcal{L} = \frac{c_1^{1-\eta}}{1-\eta} + \beta \frac{c_2^{1-\eta}}{1-\eta} + \lambda(y_2 - c_2 + (1 + R)(y_1 - c_1))$$

and the first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_1} &= c_1^{-\eta} - (1 + R)\lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial c_2} &= \beta c_2^{-\eta} - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= y_2 - c_2 + (1 + R)(y_1 - c_1) = 0. \end{aligned}$$

Rearranging the equations, we get

$$\frac{c_1}{c_2} = \left( \frac{1}{\beta(1 + R)} \right)^{1/\eta}. \quad (16)$$

To see how consumption smoothing works, consider a temporary increase in income ( $y_1$  increases but  $y_2$  remains constant). Because neither  $y_1$  or  $y_2$  show up in equation (16), the household will not change its relative consumption bundle, so that the ratio of  $c_1$  to  $c_2$  remains constant (i.e., the household smooths consumption).

This story is a little misleading for RBC theory, however, as income changes are driven by technology shocks which are typically permanent in nature. Permanent changes in income would raise  $y_1$  and  $y_2$  by equal amounts, which again does not directly affect the household's relative consumption bundle. However, with a permanent technology shock, the return to savings

$$R = MP_k = s_t(1 - \theta)k_t^{-\theta}n_t^\theta$$

will increase. We can look at three cases. (1) A household with a very low willingness to intertemporally substitute consumption ( $\eta \rightarrow \infty$ ) will choose to keep  $c_1 = c_2$  regardless of the change in  $R$ . (2) A household

with a very high willingness to intertemporally substitute consumption ( $\eta \rightarrow 0$ ) will reduce  $c_1$ , increase savings  $y_1 - c_1$ , and reap the benefits of the higher  $R$  by increasing  $c_2$ . (3) A household with ( $\eta = 1$ ) log utility and offsetting substitution and income effects will not alter savings decisions in response to  $R$ , but  $c_1/c_2$  will fall as households earn a higher return on their savings. Thus, the neoclassical model predicts consumption smoothing, and as a consequence, high investment volatility.

### 3.2 Intertemporal Substitution of Labor

In explaining labor-market fluctuations, the neoclassical model does not rely on any market failures or nominal frictions but rather on the ability and willingness of households to substitute labor across time. Consider again a two-period model with perfect foresight. Households maximize lifetime utility, written in Lagrangian form,

$$\mathcal{L} = \left\{ \ln(c_1) - \gamma \frac{n_1^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right\} + \beta \left\{ \ln(c_2) - \gamma \frac{n_2^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right\} + \lambda \left\{ w_1 n_1 + \frac{w_2 n_2}{1+R} - c_1 - \frac{c_2}{1+R} \right\}$$

by choosing consumption ( $c$ ) and labor ( $n$ ) in both periods given the wage rates  $w_1$  and  $w_2$ . Rearranging the first-order conditions produces

$$\frac{n_1}{n_2} = \left( \frac{w_1}{w_2} \beta (1+R) \right)^\sigma$$

where  $\sigma$  is the intertemporal elasticity of substitution of labor. Similar to the consumption-smoothing story above, a permanent technology shock that does not change the relative return to labor,  $w_1/w_2$ , will not directly cause an intertemporal substitution of labor. However, a positive technology shock will raise  $R$  and thus induce households to supply more labor in period one and reap the consumption benefits of the higher returns on savings. Thus the neoclassical model naturally predicts business-cycle fluctuations in hours worked in response to technology shocks.

## 4 Conclusions

For the conclusion I'll focus on two items: the title of Prescott's paper and the general arguments for and against RBC theory.

### 4.1 Prescott's Title

Why does Prescott title his paper "Theory Ahead of Business-Cycle Measurement"? To answer the question, we need to define the labor elasticity of output within the model, which is

$$\frac{\partial \ln y_t}{\partial \ln n_t} = \theta$$

and calibrated to be  $\theta = 0.64$ . Now in the actual U.S. and RBC artificial (detrended) data, if we were to run a regression of the following form

$$y_t = \theta n_t + s_t,$$

we would most certainly get an upwardly biased estimate of  $\theta$  because of the positive correlation between technology shocks ( $s_t$ ) and hours worked ( $n_t$ ). For the U.S. economy, the estimate is  $\hat{\theta} = 1.1$  and for the basic RBC model it is  $\hat{\theta} = 1.9$ . Prescott interprets the large magnitude of the U.S. estimate as "strongly supporting the importance of technology shocks in accounting for business cycle fluctuations." Furthermore, he argues that the gap between the U.S. estimate and the RBC estimate is due to measurement errors in U.S. output and states that "this deviation could very well disappear if the economic variables were measured more in conformity with theory." Hence the title.

## 4.2 Pros and Cons of RBC Theory

Here are some of the standard arguments made for and against RBC theory.

### Pros

- Simple neoclassical growth model without any money or nominal frictions can explain the comovements between and variation in several key macro variables.
- There is no room for countercyclical policy – fluctuations are optimal responses to changes in relative prices.
- Focus on microeconomic fundamentals makes the analysis less ad hoc and allows for welfare analysis.
- Avoids Lucas critique of macroeconomic policy.

### Cons

- Difficult to believe that the standard deviation of technology is nearly 1% per quarter.
- The Solow residual has been shown to be correlated with things that it shouldn't be if it were a pure measure of technology shocks.
- The intertemporal substitution of labor is small in micro studies.
- Implausible that technological regress can explain major recessions.
- Empirical studies typically show that money is non-neutral, but would be neutral in a standard RBC model.
- Calibrated models are not subject to formal hypothesis testing.
- The propagation mechanism of the RBC model is weak.