ECON 5110 Class Notes
RBC Theory: Addressing Shortcomings

1 Introduction

In this section, we examine several well-known shortcomings and modifications of RBC theory. The topics, while far from exhaustive, give a general sense of the direction that RBC research has taken since the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983).

2 Labor Market

2.1 Hansen (1985): Labor Indivisibilities

Student presentation.

2.2 Christiano and Eichenbaum (1992): Government Spending

Student presentation.

2.3 Aadland (2001): High-Frequency Real Business Cycles

This paper attempts to explain the same two labor-market puzzles addressed in Hansen (1985) and CE (1992):

- volatility of hours worked relative to output and
- correlation of real wage and hours worked.

While those papers added new features (i.e., labor indivisibilities and government spending) to the baseline RBC theory, this paper maintains the baseline theory and simply changes the decision interval. Nearly all RBC models assume that agents make decisions at a quarterly frequency (i.e., four times per year). This assumption is made so that the artificial data from the model can be easily compared with the U.S. quarterly data. However, agents likely make decisions more frequently than four times per year. In this paper, I allow agents to make decisions more frequently (e.g., once per week), temporally aggregate the artificial data up to the quarterly level, and then reassess the performance of the RBC model.

2.3.1 Benchmark Model

A representative agent is assumed to choose weekly consumption $\{c_t\}$ and hours worked $\{n_t\}$ streams to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j [(1-\phi)\log(c_{t+j}) + \phi \log(N - n_{t+j})]$$

subject to the resource constraint

$$k_t^{\theta}(a_t n_t)^{1-\theta} \geq c_t + k_{t+1} - (1-\delta)k_t$$

and the law of motion for technological change (a_t)

$$a_t = a_{t-1}\exp(\mu + \epsilon_{at}).$$

2.3.2 High-Frequency Calibration

The weekly model is calibrated by adjusting the standard quarterly parameter values using ad hoc transformation rules. Denoting the weekly parameters with asterisks, for example

- $\beta_* = \beta^{1/13} = 0.9926^{1/13} = 0.9994.$
- $N_* = N/13 = 1369/13 = 105.3.$
- $\phi_* = \phi = 2/3.$

See Table 1 for the full set of calibrated parameters.

2.3.3 Simulation and Temporal Aggregation

The calibrated RBC model is then solved and used to simulate weekly artificial data. The weekly data are then aggregated over time up to the quarterly frequency using the same procedures employed by the U.S. data collection agencies (e.g., Bureau of Economic Analysis, Bureau of Labor Statistics). A particularly important fact is that the household and establishment surveys administered by the BLS use the "calendar week that contains the 12th day of the month" as the reference period for the entire month.

The temporal aggregation operators are

$$B_{TA}(L) = 1 + L + L^2 + \cdots + L^{12}$$

for flow variables such as output and consumption and

$$B_{SS}(L) = \frac{13}{3}(L^2 + L^6 + L^{10})$$

for flow variables such as total hours worked, where L is the lag operator satisfying $L^j X_t = X_{t-j}$. The aggregation operators are more complex when the variables are measured in logarithms and first differences.

2.3.4 Comparison of the Basic and Aggregate Covariances

Since RBC theorists focus on the second-moment properties of the data (i.e., standard deviations and cross correlations), it will be useful to relate the aggregate (quarterly) and basic (weekly) covariances. The general relationship is

$$\Gamma_{xy}(0) = \omega_x \gamma_{xy}(0) \omega_y' \tag{1}$$

where

- $\Gamma_{xy}(0)$ is the zeroth-order covariance between aggregate x and aggregate y;
- ω_x is the vector describing the temporal aggregation of x;
- ω_y is the vector describing the temporal aggregation of y;
- $\gamma_{xy}(0)$ is the matrix of autocovariances between basic x and basic y.

For example, consider three-period aggregation ($n = 3$) of a flow variable x with $B_{TA}(L) = 1 + L + L^2$ and end-of-period sampling of a stock variable y with $B_{SS}(L) = 1$. Using equation (1) and assuming the variables are measured in growth rates, we get

$$\Gamma_{xy}(0) = \frac{1}{3}(\gamma_{-4} + 3\gamma_{-3} + 6\gamma_{-2} + 7\gamma_{-1} + 6\gamma_0 + 3\gamma_1 + \gamma_2).$$

Continuing with this example, imagine that x is output and y is the real wage. The RBC model predicts a large positive contemporaneous covariance, say, $\gamma_0 = 0.9$, and weaker cross covariances, say, $\gamma_j = -0.25$ for $j = -4, -3. -2, -1, 1, 2$. So even if the RBC theory were the true model, we would observe, after aggregation

$$\Gamma_{xy}(0) = \frac{1}{3}[-21(0.25) + 6(0.9)] = 0.05$$

in the U.S. data and approximately $\Gamma_{xy}(0) = 0.9$ from the standard quarterly RBC model. Therefore, we would mistakenly reject the RBC theory.

2.3.5 Results

For the complete set of results, see Table 2 on page 285. Here is a summary of the main results

Statistic	U.S. Data	Baseline RBC Model		Home Production Model	
		Quarterly	Weekly	Quarterly	Weekly
std(y)	0.96	0.96	0.96	0.96	0.96
std(n)	0.94	0.30	0.38	0.67	0.88***
corr(w,n)	-0.35	0.97	0.60	0.12	-0.30***

where (***) indicates failure to reject that the statistic is equal to that in the U.S. data. The home production model improves the performance of labor-market fluctuations because home-production technology shocks generate additional substitution between market and home activities, which also weakens the correlation between market wages and hours worked.

2.3.6 Conclusions

A strong assumption in RBC models (as well as other models of the business cycle) is that agents make decisions once per quarter. In reality, agents in our economy make economic decisions on a much more frequent basis. This paper modifies a standard RBC model to allow agents to make decisions on a weekly basis. With careful treatment of the time aggregation and sampling properties of actual U.S. data, the weekly RBC model comes closer to resolving a couple of well-known labor-market anomalies. In fact, the weekly RBC model with household production is statistically indistinguishable from the U.S. economy along these two dimensions.

3 Propagation

3.1 Cogley and Nason (1995): Output Dynamics

Cogley and Nason (1995) ask whether standard RBC models can match the stylized facts about output dynamics. Two well-known facts about U.S. output are

1. GNP growth is positively autocorrelated over short horizons and

2. GNP has a strong hump-shaped trend-reverting component.

See Figure 1, page 494.

3.1.1 Baseline RBC Model

Cogley and Nason use the CE (1992) RBC model as their baseline case. A representative agent maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j [\log(c_{t+j}) + \gamma(N - n_{t+j})]$$

subject to the resource constraint

$$y_t = k_t^{\theta}(a_t n_t)^{1-\theta} \geq c_t + k_{t+1} - (1-\delta)k_t$$

and two shocks. The technology shock follows

$$a_t = a_{t-1}\exp(\mu + \epsilon_{at})$$

and the government spending shock follows

$$\bar{g}_t = g \times \bar{g}_{t-1}^{\rho}\exp(\epsilon_{gt})$$

where $g = \exp((1-\rho)\bar{g})$ and $\bar{g}_t = g_t/a_t$. The model is calibrated as follows:

Parameter	β	γ	θ	δ	μ	$\bar{g}$	ρ	σ_a	σ_g
Value	$1.03^{-0.25}$	0.0037	0.344	0.021	0.004	0.177	0.96	0.0097	0.0113

Using Monte Carlo simulations, 1000 artificial samples are generated, each with a length of 140 quarters.

3.1.2 Autocorrelation Function (ACF) Results

Using the generalized Q statistic with an asymptotic chi-square distribution,

$$Q_{ACF} = (\hat{c} - c)'\hat{V}_c^{-1}(\hat{c} - c),$$

where $\hat{c}$ is the vector of U.S. autocorrelations, $c = \frac{1}{1000}\sum_{i=1}^{1000} c_i$, and $\hat{V}_c = \frac{1}{1000}\sum_{i=1}^{1000}(c_i - c)(c_i - c)'$, Cogley and Nason test whether the CE RBC and the U.S. output growth autocorrelations are different. CN soundly reject the null that they are the same ACFs (see Table 1 and Figure 3). In fact, the autocorrelation function in the CE RBC model is very nearly zero at all horizons.

3.1.3 Impulse Response Function (IRF) Results

Using the same generalized Q test, CN also reject that transitory and permanent IRFs from the CE RBC model are equal to those in the U.S. data (see Table 1 and Figure 3). In particular, the CE RBC model produces a damped, hump-less response to transitory shocks.

3.1.4 Propagation

In an earlier paper in *Economics Letters*, Cogley and Nason (1993) show that output in the CE RBC model can be decomposed into its permanent and transitory parts: $y(t) = y_P(t) + y_T(t)$ where

$$y_P(t) = \left[\frac{\epsilon_a(t)}{1-L}\right]\left[(1.04)\frac{1-0.87L}{1-0.94L}\right]$$

and

$$y_T(t) = \left[\frac{\epsilon_g(t)}{1-\rho L}\right](0.16).$$

The first part of each expression refers to the shock dynamics and the second, the propagation dynamics. Clearly there is no propagation mechanism for the transitory part (i.e., dampening scalar 0.16) and very little with the permanent part since $(1-0.87L)$ and $(1-0.94L)$ nearly cancel out. Overall, this implies that output in the CE RBC model inherits the dynamics of the exogenous shock processes with little additional dynamics coming from within the model. This can also be seen in the ACFs because CE RBC output growth and the growth in total factor productivity are each white noise.

3.1.5 Employment Lags and Labor Adjustment Costs

Some progress on output dynamics can be made by incorporating employment lags (e.g., labor-hoarding model of Burnside et al., 1993 in which firms choose the number of workers before observing the state of the economy and then vary work effort afterwards) and adjustment costs to changing employment. Both models do a better job of matching U.S. output dynamics, however they still require implausibly large transitory government spending shocks to match the magnitude of output dynamics.

3.1.6 Conclusions

The main contribution of Cogley and Nason (1995) is to document that standard RBC models are incapable of matching the output dynamics of U.S. GNP. This is primarily due to the absence of any endogenous propagation mechanism in the models. Therefore, output in the model basically inherits the dynamic properties of the exogenous shock processes.

4 International

4.1 Backus, Kehoe and Kydland (1992): International Real Business Cycles

Student presentation.