

# ECON 5110 Class Notes

## Staggered Wages

### 1 Introduction

A popular method for generating persistence and wage/price stickiness in current macro models is through staggered contracts. The earliest work in this area is credited to Stanley Fischer (1977) and John Taylor (1979, 1980). I will focus on the Taylor's (1979) AER article. Taylor's model is important because it presents a framework where aggregate demand disturbances can have real effects that are spread out over long periods of time (i.e., display persistence) without sacrificing rational expectations. It also produces a familiar Phillips curve tradeoff.

### 2 Taylor's (1979) Model

#### 2.1 Staggered Wage Setting

Begin by assuming that firms and workers negotiate labor contracts that specify fixed nominal wages for two periods (i.e., one year). Contracts are staggered. Half are set in January and half in July. The wage setting equation is

$$x_t = bx_{t-1} + d\hat{x}_{t+1} + \gamma(b\hat{y}_t + d\hat{y}_{t+1}) + \epsilon_t \quad (1)$$

where

- $x_t$  is the log of the contract wage in period  $t$ ;
- $b$ ,  $d$  and  $\gamma$  are positive parameters with  $b + d = 1$ ;
- $y_t$  is log excess demand (i.e., log output gap);
- a hat over a variable represents rational expectations based on time  $t - 1$  information.

Firms, unions and workers care about relative wages. Therefore, when setting wages in the beginning of period  $t$  (which will be in effect through periods  $t$  and  $t + 1$ ), agents care about wages set in period  $t - 1$  (known with certainty) and wages to be set in period  $t + 1$  (unknown in period  $t$ ). Agents also care about labor-market conditions throughout periods  $t$  and  $t + 1$ , as measured by  $y_t$ .

## 2.2 Money Demand

Money demand is taken from the quantity equation

$$m_t = y_t + w_t - v_t \tag{2}$$

where

- $m_t$  is the log of money demand;
- $w_t$  is the log of the aggregate wage level;
- $v_t$  is a velocity shock.

The aggregate wage level is assumed to follow

$$w_t = 0.5(x_t + x_{t-1}). \tag{3}$$

## 2.3 Money Supply

Money supply is given by a simple policy rule. Assume that the monetary authorities set the money supply according to

$$m_t = gw_t \tag{4}$$

where  $g$  indicates the degree of accommodation to changes in the aggregate wage level. If  $g = 0$ , then the central bank does not accommodate wage changes. If  $g = 1$ , then they accommodate them one-for-one.

## 2.4 Aggregate Demand

By equating equations (2) and (4), we get the aggregate demand curve

$$\begin{aligned} y_t + w_t - v_t &= gw_t \text{ or} \\ y_t &= -\beta w_t + v_t \end{aligned} \tag{5}$$

where  $\beta = 1 - g$ .

## 2.5 Solving for the Reduced Form

The model currently has three equations in three endogenous variables ( $w_t$ ,  $x_t$  and  $y_t$ ). The system can be collapsed down to a single linear expectational difference equation by substituting equations (5) and (3) into equation (1). This produces

$$\begin{aligned}
 \hat{x}_t &= b\hat{x}_{t-1} + d\hat{x}_{t+1} + \gamma(b(-\beta(0.5(\hat{x}_t + \hat{x}_{t-1}))) + \hat{v}_t) + d(-\beta(0.5(\hat{x}_{t+1} + \hat{x}_t)) + \hat{v}_{t+1}) + \hat{\epsilon}_t \\
 \hat{x}_t &= b\hat{x}_{t-1} + d\hat{x}_{t+1} - 0.5\gamma b\beta\hat{x}_t - 0.5\gamma b\beta\hat{x}_{t-1} - 0.5\gamma d\beta\hat{x}_{t+1} - 0.5\gamma d\beta\hat{x}_t \\
 0 &= [b - 0.5\gamma b\beta]\hat{x}_{t-1} + [-0.5\gamma b\beta - 0.5\gamma d\beta - 1]\hat{x}_t + [d - 0.5\gamma d\beta]\hat{x}_{t+1} \\
 0 &= b\hat{x}_{t-1} - c\hat{x}_t + d\hat{x}_{t+1}
 \end{aligned} \tag{6}$$

where

$$c = \frac{1 + 0.5\gamma\beta}{1 - 0.5\gamma\beta}.$$

Equation (6) is of the same form we have analyzed throughout the semester – a linear dynamic rational expectations difference equation. We have solved this type of model using repeated substitutions and using Farmer’s eigenvalue method. Another solution technique is that of undetermined coefficients (see Romer section 6.6 or Blanchard and Fischer appendix 5A). First, we guess the form of the solution (based on experience) with coefficients to be determined

$$x_t = \alpha x_{t-1} + \epsilon_t. \tag{7}$$

Using equation (7) to form expectations  $\hat{x}_t$  and  $\hat{x}_{t+1}$ , we equate coefficients to find

$$\alpha = \frac{c - [c^2 - 4d(1-d)]^{0.5}}{2d}.$$

Using equations (7) and (3), we get the reduced form for aggregate wage dynamics

$$\begin{aligned}
 w_t &= \alpha w_{t-1} + 0.5(\epsilon_t + \epsilon_{t-1}) \\
 &= 0.5 \sum_{i=0}^{\infty} \alpha^i (\epsilon_{t-i} + \epsilon_{t-1-i})
 \end{aligned}$$

and the reduced form for output dynamics

$$\begin{aligned} y_t &= \alpha y_{t-1} - 0.5\beta(\epsilon_t + \epsilon_{t-1}) + (v_t - \alpha v_{t-1}) \\ &= -0.5\beta \sum_{i=0}^{\infty} \alpha^i (\epsilon_{t-i} + \epsilon_{t-1-i}) + v_t. \end{aligned}$$

## 2.6 Interpreting the Results

### 2.6.1 Phillips Curve Tradeoff

Output and wage dynamics are governed by the parameter  $\alpha$ . Lower values of  $\alpha$  lead to less persistence (i.e., higher stability) in aggregate wages. Note also that  $g$  and  $\alpha$  are positively related. Therefore, the central bank can generate more stability in aggregate wages by making  $g$  small. However, this comes at an expense. Lower values of  $g$  imply higher values of  $\beta$  and thus a flatter aggregate demand curve. This means that shocks to contract wages will lead to more output volatility. The tradeoff exists in reverse if the central bank is more accommodative (i.e., makes  $g$  large).

The Phillips curve tradeoff can be summarized as follows:

- Accommodative policy ( $g$  high,  $\beta$  low)  $\longrightarrow$  low variation in  $y$  and high variation in  $w$ ;
- Not accommodative policy ( $g$  low,  $\beta$  high)  $\longrightarrow$  high variation in  $y$  and low variation in  $w$ .

### 2.6.2 Degree of Forward-Looking Behavior

It is interesting to see how wage and output dynamics depend on the degree of forward-looking behavior. As expected, wage persistence (as measured by  $\alpha$ ) is decreasing in  $d$ . This makes intuitive sense. Consider the limiting case of  $d = 1$ , where agents only look forward to next period's wage. In this case,  $\alpha = 0$  and a shock to the contract wage only lasts as long as the contract period. On the other hand, when  $d = 0$  and agents only look backwards,  $\alpha$  is very nearly one and wages are very persistent. In sum,

- Only forward-looking behavior ( $b = 0, d = 1, \alpha = 0$ )  $\longrightarrow$  low persistence in  $w$  and  $y$ ;
- Only backward-looking behavior ( $b = 1, d = 0, \alpha \simeq 1$ )  $\longrightarrow$  high persistence in  $w$  and  $y$ .

### 2.6.3 Hump-Shaped Impulse Response Functions

A well-known feature of U.S. output is that it displays a hump-shaped impulse response that peaks somewhere in the neighborhood of four quarters. For example, if we let  $b = d = 0.5, \gamma = 0.2, g = 0.5$  and  $v_t = 0$  for all

$t$ , we get the following equation for output dynamics

$$y_t = 0.635y_{t-1} - 0.25(\epsilon_t + \epsilon_{t-1}),$$

which is an ARMA(1,1) process. Assuming  $y_0 = \epsilon_0 = 0$ ,  $\epsilon_1 = -1$  and  $\epsilon_t = 0$  for all  $t > 1$ , we get the following sequence for output:

period ( $j$ )	$y_j$
0	0
1	0.25
2	0.41
3	0.26
4	0.17

Therefore, a one-time shock to the contract wage produces a hump-shaped response for output that peaks at four quarters (i.e., one year) after the shock.

### 3 Menu Costs

Student presentation.