1 Introduction

In this section, I present an overview of business-cycle models that are driven by extrinsic or non-fundamental uncertainty. Typical business-cycle models are driven by intrinsic or fundamental uncertainty (i.e., uncertainty related to preferences, technology, endowments). In these models, intrinsic uncertainty is often assumed to arise from technology, government spending, or monetary shocks. Extrinsic uncertainty, on the other hand, is anything that is unrelated to economic fundamentals. Keynes’ animal-spirits hypothesis was an early application of the idea that the business cycle might be driven by extrinsic uncertainty. The research program advocated in Farmer’s *Macroeconomics of Self-Fulfilling Prophecies*, which I rely on heavily in this section, can be considered a modern formalization of Keynes’ animal-spirits hypothesis.

2 Classification of Business-Cycle Models

Most modern business-cycle models can be written as a system of nonlinear, expectational difference equations

\[ y_t = E_t f(y_{t+1}, x_t) \]  

where \( y_t \) is a vector of endogenous variables (e.g., prices, output) and \( x_t \) is a vector exogenous or predetermined variables (e.g., government spending, money supply, technology shocks). The expectation operator \( E_t \) is typically assumed to represent rational expectations conditioned upon all known information at time \( t \) and earlier. The rational expectations error \( \epsilon_{t+1} = w_{t+1} - E_t(w_{t+1}) \) obeys \( E_t(\epsilon_{t+1}) = 0 \). To solve these types of models, one generally requires that (1) be linearized

\[ y_t = bE_t y_{t+1} + cx_t. \]  

We divide the system (2) into two classes depending on the value of \( b \).

2.1 Regular Case

The regular case is defined by \(|b| < 1\). Most models of the business-cycle fall into the regular class. These models have unique rational expectation equilibria, or in other words, a single convergent path to the steady-state equilibrium. Prominent examples include the RBC model and Taylor’s overlapping wage model.
2.1.1 An Example. Cagan’s Inflation Model

Cagan’s inflation model begins with a simple money demand function

\[ m_t - p_t = -\alpha(E_t p_{t+1} - p_t) \]  \hspace{1cm} (3)

where \( \alpha > 0 \), \( m_t \) is the log of the nominal money balances (exogenous), and \( p_t \) is the log of the price level (endogenous). Notice that this money demand function ignores income and interest rates, which is reasonable in times of high inflation. Rearranging (3) into the form of (2) gives

\[ p_t = bE_t p_{t+1} + cm_t \]

where \( b = \alpha/(1 + \alpha) \) and \( c = 1/(1 + \alpha) \). Because \( 0 < b < 1 \), this is an example of a regular model.

2.1.2 A Special Case

Now consider a special case of the more general model (2) where \( x_t = x \) and \( E_t y_{t+1} = y_{t+1} \). The model is

\[ y_t = by_{t+1} + cx \Rightarrow \]
\[ y_{t+1} = b^{-1}y_t - b^{-1}cx. \]  \hspace{1cm} (4)

Since we are discussing the regular case, we know that \( b^{-1} > 1 \) and there exists only one non-explosive solution

\[ \bar{y} = \frac{cx}{1-b}. \]

2.1.3 A Less Special Case

We now relax the assumption of a nonstochastic \( x_t \) and perfect foresight. Let expectations be rational and \( x_t \) be a stationary stochastic process. Begin with

\[ y_t = bE_t y_{t+1} + cx_t \]

and substitute for \( y_{t+1} \)

\[ y_t = bE_t[bE_{t+1}y_{t+2} + cx_{t+1}] + cx_t. \]

Using the law of iterated expectations, we get

\[ y_t = b^2E_t y_{t+2} + (cx_t + bE_t x_{t+1}). \]
Further substitutions produce
\[ y_t = b^s E_t y_{t+s} + \sum_{i=0}^{s-1} b^i c E_t x_{t+i}. \]

Letting \( s \to \infty \) and assuming \( E_t y_{t+s} \) does not explode too fast,
\[ y_t = c \sum_{i=0}^{\infty} b^i E_t x_{t+i} = F_t \]
because \(|b| < 1\). We will refer to \( F_t \) as the fundamental solution. It is useful to verify that \( F_t \) is indeed a solution by substituting it back into (2)
\[
\begin{align*}
y_t &= b E_t y_{t+1} + c x_t \\
c \sum_{i=0}^{\infty} b^i E_t x_{t+i} &= b E_t \left[ c \sum_{i=0}^{\infty} b^i E_{t+1} x_{t+1+i} \right] + c x_t \\
&= c \sum_{i=0}^{\infty} b^{i+1} E_t x_{t+1+i} + c x_t \\
&= c \sum_{i=0}^{\infty} b^i E_t x_{t+i}.
\end{align*}
\]

This verifies that \( F_t \) is a solution, but is it the only one?

### 2.1.4 Bubbles

Next, we look for solutions of the form \( y_t = F_t + B_t \). Substitution into (2) gives
\[
F_t + B_t = b(E_t F_{t+1} + E_t B_{t+1}) + c x_t.
\]
Since \( F_t = b E_t F_{t+1} + c x_t \), this implies that in order for \( y_t = F_t + B_t \) to be a solution, we require
\[
B_t = b E_t B_{t+1}. \tag{5}
\]

\( B_t \) that satisfy equation (5) are often referred to as rational or speculative bubbles.

**Examples of Bubbles** Here are three types of bubbles.

1. **Deterministic, Ever-Expanding Bubble.**
   \[
   B_t = b^{-1} B_{t-1}
   \]
   where \(|b| < 1\) and \( B_0 \) is given. This bubble satisfies equation (5).

2. **Stochastic Bubble.**
   \[
   B_t = b^{-1} B_{t-1} z_t
   \]
where $z_t > 0$ is an i.i.d. stochastic process satisfying $E_t z_{t+1} = 1$. This bubble satisfies equation (5).

3. Periodically Popping Bubble.

$$B_t = \frac{(B_{t-1} - B_s)(b \pi)^{-1}}{B_s (b(1 - \pi))^{-1}} \quad \text{with probability } \pi$$

This bubble satisfies equation (5).

**Can We Rule Rational Bubbles Out?** Sometimes we can and sometimes we can’t. For example, bubbles cannot be ruled out on the price of an intrinsically worthless asset that goes on into perpetuity, especially one for which fundamentals are hard to pin down. However, bubbles can be ruled out on assets with known finite maturity or if their value becomes so large that they are not consistent with the notion of finite resources. Note that all bubbles are explosive:

$$B_t = b E_t B_{t+1}$$

$$= b E_t [b E_{t+1} B_{t+2}]$$

$$= b^2 E_t B_{t+2}$$

$$\vdots$$

$$B_t = b^s E_t B_{t+s} \Rightarrow$$

$$E_t B_{t+s} = b^{-s} B_t.$$

Every rational bubble therefore satisfies $\lim_{s \to \infty} E_t B_{t+s} = \infty$ if $B_t > 0$. Since we are analyzing economies comprised of a fixed number of infinitely lived agents, rational bubbles can be ruled out. Agents purchase the asset at a price higher than indicated by fundamentals only with the expectation of selling it for a gain in the future. Without new entrants into the economy, it cannot be an equilibrium for everyone to do this. In this sense, the bubble is similar to a Ponzi game.

### 2.2 Irregular Case

The irregular case is defined by $|b| > 1$. These models allow for multiple nonexplosive solutions, that is, **multiple equilibria**. As you will see, this class of model can support the notion of animal spirits or self-fulfilling prophecies.
2.2.1 A Special Case

As before, consider the special case of perfect foresight and nonstochastic $x$. The system can be written as equation (4)

$$y_{t+1} = b^{-1}y_t - b^{-1}cx$$

where $|b^{-1}| < 1$. Therefore, this linear difference equation is stable and there exist an infinite number of equilibrium paths that converge on the steady state $\bar{y} = cx/(1-b)$.

2.2.2 Complete Class of Solutions

To analyze the complete class of solutions, define $\eta_{t+1} = y_{t+1} - E_t y_{t+1}$, where the definition of rational expectations gives $E_t \eta_{t+1} = 0$. Substitution into (2) and letting $x_t = x$, we have

$$y_t = b(y_{t+1} - \eta_{t+1}) + cx \Rightarrow$$

$$y_{t+1} = -b^{-1}cx + b^{-1}y_t + \eta_{t+1}.$$  (6)

Equation (6) is a stationary first-order autoregressive (AR(1)) process where $\eta_{t+1} \sim iid(0, \sigma^2_\eta)$.

Notes.

- $\eta_t$ has many names:
  - extrinsic noise
  - nonfundamental noise
  - sunspot
  - self-fulfilling prophecy
  - animal spirits
- The $B_t$ bubbles in the regular case can be thought of as explosive sunspots.
- There are two sources of indeterminacy – $y_0$ and $\eta_t$.
- $\eta_t$ can be any variable coordinating expectations and matters only because people believe it does.
- Jevons (1884) thought sunspots actually were affecting the economy.

2.2.3 Other Types of Multiplicity

There are other types of multiple equilibrium that do not fall into the regular or irregular class.

1. Regular cycles. For some $y_t = f(y_{t+1}, x_t)$, it may be possible to generate, say, regular two-cycles or three-cycles.
2. Multiple steady states. Other $y_t = f(y_{t+1}, x_t)$ may produce multiple steady states with the possibility that sunspots may jump you from one steady state to another. Models of this type could be used to look at coordination failures.

3. Chaos. Still other $y_t = f(y_{t+1}, x_t)$ may exhibit chaotic behavior where small changes in the initial conditions may produce a rich variation in dynamic behavior.

2.3 Overlapping Generations (OG) Example

Economists have long recognized that dynamic rational expectations models may exhibit multiple equilibria. We will discuss below how this may occur in an infinitely lived representative agent (RA) model. However, the best known examples come from overlapping generations (OG) models. Here’s one example.

**Framework.**
- one good
- constant stock of money, $M$
- two-period lives
- no production
- constant population – half young, half old
- endowment – $(e_1, 0)$
- $P_t$ is the money price of the good

The objective for agents is to choose consumption in period one and two $(c_{1,t}, c_{2,t+1})$ to maximize

$$V(c_{1,t}) + W(c_{2,t+1})$$

subject to

$$c_{1,t} + \frac{M}{P_t} = e_1 \quad \text{and} \quad c_{2,t+1} = \frac{M}{P_{t+1}}.$$ 

Putting the constraints together gives

$$c_{2,t+1} = (e_1 - c_{1,t}) \frac{P_t}{P_{t+1}}.$$ 

(8)

where $P_t/P_{t+1}$ is the gross rate of return on holding money. Substituting (8) into (7) gives

$$V(c_{1,t}) + W((e_1 - c_{1,t}) \frac{P_t}{P_{t+1}}).$$
The first-order condition is
\[
\frac{\partial (\bullet)}{\partial c_{1,t}} = V'(c_{1,t}) - \frac{P_t}{P_{t+1}} W'((e_1 - c_{1,t}) \frac{P_t}{P_{t+1}}) = 0
\]
which after rearranging gives
\[
V'(c_{1,t}) = \frac{P_t}{P_{t+1}} W'(e_{2,t+1}). \tag{9}
\]
Although \( V \) and \( W \) in (9) do not have explicit functional forms, we can imagine solving implicitly for first-period consumption demand
\[
c_{1,t} = c_1(e_1, \frac{P_t}{P_{t+1}}).
\]
Plugging this back into the first-period constraint gives
\[
\frac{M}{P_t} = e_1 - c_1(e_1, \frac{P_t}{P_{t+1}}) = L(e_1, \frac{P_t}{P_{t+1}}), \tag{10}
\]
which is a demand function for real money balances. The derivative of money demand with respect to \( e_1 \) will be unambiguously nonnegative (i.e., extra endowment in period one will not cause the agent to want less consumption in period two). The derivative with respect to the rate of return on money, however, could be positive or negative. Imagine a small decrease in \( P_{t+1} \). If the substitution effect dominates, then a higher \( P_t/P_{t+1} \) will induce the agent to substitute consumption tomorrow for consumption today and therefore demand more money (i.e., \( L_2 > 0 \)). If the income effect dominates, then a higher \( P_t/P_{t+1} \) increases wealth and period-one consumption demand (i.e., \( L_2 < 0 \)). Next, we linearize (10) since it is likely to be nonlinear
\[
m - p_t = \gamma(e_1) + \alpha(p_t - E_t p_{t+1})
\]
where lower-case letters represent proportional deviations from the steady state. This can be rearranged into our standard form as
\[
p_t = a + b E_t p_{t+1} \tag{11}
\]
where \( b = \alpha/(1 + \alpha) \) and \( a = (m - \gamma(e_1))/(1 + \alpha) \). There are two cases.

1. Regular case. In the regular case, \( |b| < 1 \) and the substitution effect dominates the income effect. This means that \( \alpha > -0.5 \). The unique solution, which can be found by repeated substitutions into the right-hand side of (11), is
\[
p_t = \frac{a}{1 - b}.
\]

2. Irregular case. In the irregular case, \( |b| > 1 \) and the income effect dominates the substitution effect. This means that \( \alpha < -0.5 \). The full set of solutions can be found by substituting \( \eta_{t+1} = p_{t+1} - E_t p_{t+1} \)
into (11) and rearranging
\[ p_t = -ab^{-1} + b^{-1} p_{t-1} + \eta_t \]

where \( \eta_t \) is a sunspot. An interesting special case occurs when \(-1 < \alpha < -0.5\), producing the so-called "cobweb model".

2.3.1 Parametric Example

Let the utility functions be CES
\[ V(c_{1,t}) = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta}; \quad W(c_{2,t+1}) = \beta \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta} \]

and
\[ V'(c_{1,t}) = c_{1,t}^{-\theta}; \quad W'(c_{2,t+1}) = \beta c_{2,t+1}^{-\theta}. \]

Substitution into (9) with some light algebra gives
\[ \left( \frac{c_1}{c_{1,t}} - 1 \right) = \beta^{1/\theta} \left( \frac{P_t}{P_{t+1}} \right)^{\frac{1-\theta}{\theta}}. \]

Let’s look at the three cases.

1. **Regular case.** If \( 0 < \theta < 1 \), then \( c_{1,t} \) falls when \( P_t/P_{t+1} \) increases. In other words, when today’s price increases (relative to tomorrow), you consume less today (save more). The substitution effect dominates.

2. **Irregular case.** If \( \theta > 1 \), then \( c_{1,t} \) rises when \( P_t/P_{t+1} \) increases. In other words, when tomorrow’s price decreases (relative to today), you consume more today because you are wealthier. The income effect dominates.

3. **In-Between case.** If \( \theta = 1 \), then utility is log in consumption and \( c_{1,t} \) does not depend on relative prices. The substitution and income effects cancel one another out.

3 Multiple Equilibria Based on Increasing Returns (IR)

While OG models can exhibit multiple equilibria, they are difficult to implement empirically. Rather, Farmer uses the standard RA framework of the RBC model, but adds increasing returns (IR) to scale. The problem with incorporating IR into the neoclassical model is that it is inconsistent with competitive behavior. However, it is possible to reconcile the neoclassical model with IR by modifying the model appropriately. Here are two approaches.
3.1 Externalities Approach

Let the representative agent be indexed by $i$ on the unit interval $[0, 1]$. The $i^{th}$ agent chooses consumption and labor $(l_i, t)$ to maximize

$$U_i = \sum_{s=t}^{\infty} E_t \beta^{t-s} \left[ \log(c_{is}) - \frac{j_i}{1 - \chi} \right]$$

subject to

$$c_{i,t} + k_{i,t+1} = k_{i,t}(1 - \delta) + y_{i,t} \text{ and}$$

$$y_{i,t} = A_t s_t k_m^m(\gamma^t l_i, t)^{1-m}$$

where $A_t$ is an aggregate production externality (organizational synergies) given by

$$A_t = \left[ \int_0^1 k_{i,t}^m(\gamma^t l_i, t)^{1-m} di \right]^\theta.$$ 

Recognizing that all agents are identical allows us to write aggregate output as

$$y_t = \int_0^1 y_{i,t} di = \int_0^1 A_t s_t k_m^m(\gamma^t l_i, t)^{1-m} di$$

$$= s_t A_t \int_0^1 k_m^m(\gamma^t l_i, t)^{1-m} di$$

$$= s_t \left[ \int_0^1 k_m^m(\gamma^t l_i, t)^{1-m} di \right]^{\theta} \int_0^1 k_m^m(\gamma^t l_i, t)^{1-m} di$$

$$= s_t \left[ k_m^m(\gamma^t l_i)^{1-m} \right]^{\theta + m}$$

$$= s_t k_m^m(\gamma^t l_i)^{1 + \theta}$$

where $\mu = m(1 + \theta)$, $\nu = (1 - m)(1 + \theta)$ and $\mu + \nu > 1$. Hence we have constant returns to scale at the private level and IR at society’s level. When making private decisions, agents will take $A_t$ as given and each factor will be paid its private marginal product, which in aggregate will exhaust all of national income. There is no inconsistency between competitive markets and IR at the social level.

3.2 Monopolistic Competition Approach

Under the monopolistic competition approach, begin by assuming that each agent produces a distinct intermediate good with an IR production technology. The intermediate goods are aggregated in a competitive
sector to form a final good using the following technology

\[ y_t = \left[ \int_i y_t^\lambda di \right]^{1/\lambda}. \]  (12)

### 3.2.1 Final-Goods Producers

Each final-goods producer then chooses \( y_{i,t} \) for \( i \in [0, 1] \) to maximize

\[ \pi = p_t y_t - \int_i p_{it} y_{it} di \]  (13)

subject to (12). Substituting (12) into (13) produces

\[ \pi = p_t \left[ \int_i y_t^\lambda di \right]^{1/\lambda} - \int_i p_{it} y_{it} di. \]

Taking the derivative with respect to \( y_{it} \), setting it equal to zero, letting \( p_t = 1 \) be the numeraire, and rearranging gives the demands for intermediate goods

\[ p_{it} = \left( \frac{y_{it}}{y_t} \right)^{\lambda-1}. \]  (14)

Therefore, if \( \lambda = 1 \) final output is a simple sum of all intermediate goods (i.e., they are perfect substitutes), \( p_{it} = 1 \) and intermediate-goods producers are price takers. If \( \lambda < 1 \), the \( y_{it} \)'s are imperfect substitutes and there is some market power in the intermediate sector.

### 3.2.2 Intermediate-Goods Producers

Intermediate-goods producers, taking (14) as given, choose \( l_{it} \) and \( k_{it} \) to maximize

\[ \pi_{it} = \left( \frac{y_{it}}{y_t} \right)^{\lambda-1} y_{it} - w_t l_{it} - r_t k_{it} \]  (15)

where

\[ y_{it} = s_t k_{it}^{\mu} (\gamma_{it} t_{it})^{\nu} \]  (16)

with \( \mu + \nu > 1 \). Substitution of (16) into (15), gives

\[ \pi_{it} = y_t^{1-\lambda} \left[ s_t^\lambda k_{it}^{\mu \lambda} (\gamma_{it} t_{it})^{\nu \lambda} \right] - w_t l_{it} - r_t k_{it} \]
where we assume that $\lambda(\mu + \nu) \leq 1$. The first-order conditions give the factor demands

$$
\begin{align*}
  r_t &= \lambda \gamma_t p_t k_{t+1}^{\lambda-1} \\
  w_t &= \lambda \gamma_t p_t l_{t+1}^{\lambda-1}.
\end{align*}
$$

3.2.3 Consumers

Consumers are assumed to choose $c_t$ and $l_t$ to maximize

$$
U_t = \sum_{s=t}^{\infty} E_t \beta^{t-s} \left[ \log(c_{is}) - \frac{k_{is}^{1-\chi}}{1 - \chi} \right]
$$

subject to the standard budget constraint. The consumers’ Euler equations are

$$
\begin{align*}
  \frac{1}{c_{it}} &= \beta E_t \left[ \frac{1}{c_{i,t+1}} (1 - \delta + r_{t+1}) \right] \quad \text{and} \\
  w_t &= c_{it} l_{it}^{1+\lambda}.
\end{align*}
$$

3.2.4 Combine Sectors

Now we impose symmetry across all consumers and firms so that in equilibrium $k_{it} = k_{jt} \equiv k_t$, $l_{it} = l_{jt} \equiv l_t$, and $p_{it} = p_{jt} = 1$, where the last equality comes from imposing zero profits ($\pi_t^* = 0$) in the competitive final-goods sector. This produces

$$
\begin{align*}
  \frac{1}{c_t} &= \beta E_t \left[ \frac{1}{c_{t+1}} (1 - \delta + \gamma_t k_{t+1}^{\lambda-1}) \right] \\
  \lambda \gamma_t &= c_t l_{t+1}^{1+\chi}.
\end{align*}
$$

3.3 Overall Increasing Returns Model

We now have two economic environments that generate increasing returns at the social level – aggregate production externalities and monopolistic competition. Although the underlying structural parameters have different interpretations, they can be written in one common framework:

$$
\begin{align*}
  k_{t+1} &= y_t + (1 - \delta)k_t - c_t \quad \text{(capital accumulation)} \\
  y_t &= s_t k_t^\gamma (\gamma_t l_t)^\nu \quad \text{(production technology)} \\
  n y_t &= c_t l_{t+1}^{1+\chi} \quad \text{(consumption-leisure tradeoff)} \\
  \frac{1}{c_t} &= \beta E_t \left[ \frac{1}{c_{t+1}} (1 - \delta + m y_{t+1} k_{t+1}^{\lambda-1}) \right] \quad \text{(consumption tradeoff)} \\
  s_t &= s_{t-1} v_t. \quad \text{(technology shock)}
\end{align*}
$$
where $\mu + \nu > 1$. The parameters are related according to

Externalities approach.

- $m =$ private share in capital
- $n =$ private share in labor
- $m + n = 1$
- $\mu = m(1 + \theta)$
- $\nu = (1 - m)(1 + \theta)$

Monopolistic competition approach.

- $m = \lambda \mu$
- $n = \lambda \nu$
- $m + n < 1$ so there are positive profits.

3.4 Empirical Evidence for Increasing Returns

A well-known puzzle in the macro literature is that of procyclical productivity in the U.S. data. Since (detrended) output is more volatile than (detrended) hours worked, it implies that average labor productivity $(y_t/n_t)$ increases when $y_t$ increases. Using this as background, we can list the following pieces of evidence in favor of IR:

- Increasing returns can solve the procyclical productivity puzzle (but of course so can the RBC model).
- The Solow residual in the RBC model is correlated with things it shouldn’t be (e.g., military expenditures).
- Econometric estimates of the marginal product of labor in larger structural models often exceed one.
- A simple regression of detrended output on detrended hours has a slope greater than one.

3.5 Comparing the RBC and IR Models

In either the case of the RBC or the IR economy, the linearized system can be written as

$$
\begin{bmatrix}
\dot{k}_t \\
\dot{c}_t \\
\dot{s}_t 
\end{bmatrix} = J \begin{bmatrix}
\dot{k}_{t+1} \\
\dot{c}_{t+1} \\
\dot{s}_{t+1} 
\end{bmatrix} + R \begin{bmatrix}
\dot{v}_{t+1} \\
\dot{w}_{t+1} 
\end{bmatrix}
$$

(17)
where \( w_{t+1} \) is an expectational error (see class notes on Solution Techniques for solving such dynamic rational expectations general equilibrium models). The models differ fundamentally depending on the nature of \( J \).

To help compare the RBC and IR models, consider the following definitions:

- \( n_s = \) number of forward stable roots
- \( n_1 = \) number of predetermined variables
- \( n_2 = \) number of free variables
- \( n = n_1 + n_2 \)

A model is regular if \( n_s = n_2 \). A model is irregular if \( n_s < n_2 \).

### 3.5.1 RBC Economy

The standard RBC economy (with \( m = \mu \) and \( n = \nu \)) has one forward-stable root \( (n_s = 1) \) and from equation (17) above, consumption is the only free variable \( (n_2 = 1) \). Therefore, the RBC economy is a regular economy exhibiting saddle-path stability. Note that \( s_t \) and \( k_t \) are predetermined variables associated with initial conditions \( s_0 \) and \( k_0 \).

### 3.5.2 IR Economy

The IR economy, on the other hand, has no forward stable roots \( (n_s = 0) \). Therefore, \( n_s < n_2 \) and it is an irregular economy. As a result, the economy will display expectational indeterminacy so that sunspots may influence the equilibrium path of the economy.

### 3.5.3 Empirical Comparison of the RBC and IR Economies

Farmer compares the RBC and IR economies by calibrating each economy. Each economy is specified to have a single shock process – technology in the RBC economy and self-fulfilling beliefs (sunspots) in the IR economy. See Farmer, Table 7.1 for details. The shock processes are given a standard deviation so that the volatility of artificial output matches that in the U.S. data.

**Relative Volatilities** The relative volatilities of the RBC and IR economies are similar in their ability to match the U.S. data, although IR consumption tends to be smoother and IR investment more volatile than the counterparts from the RBC economy. See Farmer, Table 7.2.

**Impulse Response Functions** Farmer points out that for the U.S. economy, the dynamical system (17) is best represented with complex roots so that the economy exhibits cyclical dynamics. The typical RBC economy has only real roots, while the IR economy tends to have complex roots. Therefore, the IR economy is better able to match the apparent cyclical dynamics in the U.S. economy. See Farmer, Figure 7.4.