

# Econ 5110 Midterm Exam

Spring 2008

1. Ramsey Growth Model (50 pts). Consider a Ramsey economy where agents have log utility, technology grows at rate  $g$ , and labor supply is fixed. The following system of differential equations describe the dynamics of this Ramsey economy

$$\begin{aligned}\frac{\dot{c}(t)}{c(t)} &= f_k(k(t)) - \rho - g \\ \dot{k}(t) &= f(k(t)) - c(t) - gk(t),\end{aligned}$$

where  $f(k(t)) = k(t)^\alpha$ ,  $k(0)$  is given, and all other notation is the same as in class.

- (a) (12.5 pts) Find the steady-state level of capital,  $k^*$ .
- (b) (12.5 pts) Draw the phase diagram for the dynamics of  $c$  and  $k$ . Make sure to carefully label the axes, phase arrows and saddle path.
- (c) (12.5 pts) Find the golden-rule level of capital,  $k_g$ , and provide some intuition for the difference between  $k^*$  and  $k_g$ .
- (d) (12.5 pts) Find the effects of a small change in the discount rate,  $\rho$ , on the steady-state savings rate,  $s^* = [f(k^*) - c^*]/f(k^*)$ . Does the sign on this derivative make intuitive sense? Explain.
- (e) **BONUS** (10 pts) Consider the effects of an increase in government spending (i.e., “the surge”) for the war in Iraq. Modify the differential equations above and show in the phase diagram how this is predicted to impact the economy.

2. Real Business Cycle Theory (50 pts). Consider a neoclassical growth model (notation similar to class)

where all agents are identical and a representative agent maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \{g(c_t) + \phi \ln(l_t)\}$$

by choosing  $\{c_t, l_t\}_{t=0}^{\infty}$  subject to

$$c_t + k_{t+1} \leq (1 - \delta)k_t + y_t$$

where  $g(c_t) = \ln(c_t)$ ,  $y_t = s_t k_t^{1-\alpha} n_t^\alpha$ ,  $l_t = 1 - n_t$ ,  $s_t = s_{t-1}^\rho \exp(\epsilon_t)$ ,  $\epsilon_t \sim iid(0, \sigma^2)$  and  $(s_0, k_0)$  given.

- (a) (12.5 pts) Calculate the Euler equation for consumption,  $c_t$ . Provide some economic intuition for the equation.
- (b) (12.5 pts) Describe the time paths of  $k$ ,  $c$ ,  $y$  and  $n$  to a one-time shock to  $\epsilon_t$  when  $\rho = 0.95$ . Repeat with  $\rho = 0$ . Highlight the differences using a pair of stylized impulse response functions.
- (c) (12.5 pts) The standard RBC model fails to produce sufficient variation in total hours worked to match the historical U.S. data. Describe three modifications to the basic model that will bring theory closer to the data.
- (d) (12.5 pts) Calculate the Euler equation for consumption when  $g(c_t) = c_t - \theta c_t^2$ . Are there any necessary restrictions on  $\theta$  to make this a well-defined problem? Explain.
- (e) **BONUS** (10 pts) Imagine that manufacturing output contributed to the aggregate stock of an airborne pollutant that negatively impacted the well-being of households. How would you incorporate this feature into the RBC model? Discuss the equilibrium outcome of a decentralized economy versus that of a benevolent social planner.