1. **Modified Cobweb Model.** (60 pts) Let the demand for a good \( d \) be given by

\[
d_t = \alpha_0 - \alpha_1 p_t + \alpha_2 y_t + \alpha_3 E_t y_{t+1} + \epsilon_t^d; \tag{Demand}
\]

supply \( s \) be given by

\[
s_t = \beta_0 + \beta_1 E_{t-2} p_t + \epsilon_t^s, \tag{Supply}
\]

where \( p \) is price; income \( y \) is exogenous and follows a mean-zero first-order autoregressive process with persistence parameter \( \lambda_y \); \( E_{t-j} \) is the expectations operator conditional on time \( t-j \) information; \( \epsilon_t^d \) and \( \epsilon_t^s \) are mean-zero, mutually independent white-noise error terms; all \( \alpha \)'s and \( \beta \)'s are positive; and the market clears (i.e., \( d_t = s_t \) for all \( t \)).

(a) (10 pts) Provide an economic interpretation for the two modifications to the basic cobweb model.

(b) (10 pts) Find the steady state.

(c) (10 pts) Find the equilibrium under naïve expectations. Describe the transition dynamics in words and using a diagram.

(d) (10 pts) Find the fundamental rational expectations equilibrium (REE).

(e) (10 pts) Is a rational bubble possible in equilibrium?

(f) (10 pts) Under what conditions is the REE stable under least squares learning?

2. **Dynamic New Keynesian (DNK) Model.** (40 pts) Consider the following DNK model,

\[
\begin{align*}
x_t & = -\varphi [i_t - E_t \pi_{t+1}] + \kappa E_t x_{t+1} + \epsilon_t \quad \text{(IS curve)} \\
\pi_t & = \lambda x_t + \gamma E_t \pi_{t+1} + \mu_t \quad \text{(Phillips curve)} \\
i_t & = i + \theta_x E_t x_{t+1} + \theta_\pi E_t \pi_{t+1} \quad \text{(Taylor rule)}
\end{align*}
\]

where the variable definitions are the same as those discussed in class.

(a) (10 pts) Discuss the relationship between the intertemporal elasticity of substitution for consumption and the slope of the IS curve.

(b) (10 pts) Solve for the REE under the special case where \( \kappa = \theta_x = 0 \). Use this for parts (c) and (d).

(c) (10 pts) Discuss the Taylor principle and how it relates to determinacy of the REE. How aggressive does the central bank need to be against inflation to avoid an indeterminate REE?

(d) (10 pts) Calculate the condition under which the liquidity trap will bind in the REE.