1 Two Variable Regression: Interval Estimation and Hypothesis Testing

Interval Estimation

- $\hat{\beta}_2$ value is a “point” estimate
- $(\hat{\beta}_2 - \delta, \hat{\beta}_2 + \delta)$ is an “interval” estimate with $1 - \alpha$ confidence level
- Significance vs. confidence level
- $\Pr(\hat{\beta}_2 - \delta \leq \beta_2 \leq \hat{\beta}_2 + \delta) = 1 - \alpha$
- Need a probability distribution!

Confidence Intervals

- Standard normal test statistic:
  $$Z = (\hat{\beta}_2 - \beta_2)/se(\hat{\beta}_2), \text{ where } se(\hat{\beta}_2) = \sigma/\sqrt{\sum_i(X_i - \bar{X})^2}$$
  - ...but, $\sigma$ is unknown
- Student’s $t$ test statistic:
  $$t = (\hat{\beta}_2 - \beta_2)/\hat{se}(\hat{\beta}_2), \text{ where } \hat{se}(\hat{\beta}_2) = \hat{\sigma}/\sqrt{\sum_i(X_i - \bar{X})^2}$$
  - $\Pr(-t_{\alpha/2} \leq t \leq t_{\alpha/2}) = 1 - \alpha$
  - ...and after some light algebra...
- $100(1 - \alpha)\%$ confidence level for $\beta_2$ is $\hat{\beta}_2 \pm t_{\alpha/2} se(\hat{\beta}_2)$
- Chi-square test statistic:
  $$\chi^2 = (n - 2)\hat{\sigma}^2/\sigma^2$$
  - ...and after some more light algebra, the confidence interval for $\sigma^2$ is...
- $\Pr \left[ (n - 2)\frac{\hat{\sigma}^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq (n - 2)\frac{\hat{\sigma}^2}{\chi^2_{1 - \alpha/2}} \right] = 1 - \alpha$
Hypothesis Testing

- Confidence interval vs. test-of-significance approach

- Steps in the standard approach:
  - Step #1. Form null and alternative hypotheses
  - Step #2. Choose significance level
  - Step #3. Form test statistic & identify distribution
  - Step #4. Form the decision rule
  - Step #5. Draw conclusion
  - Step #6. Consider possible errors

- $p$ values

- Statistical vs. economic significance

- Analysis of Variance (ANOVA) and the $F$ test

- Prediction
  - $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_0$
  - Confidence interval: $[\hat{Y} - t_{\alpha/2} \text{se}(\hat{Y}), \hat{Y} + t_{\alpha/2} \text{se}(\hat{Y})]$

- Reporting regression results