ECON 5350 Solutions to Problem Set #4

1. Greene 5th edition, Exercise 4.3.

   Answer: The variance of the least squares slope estimator with a constant is

$$var(b) = \frac{\sigma^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

   and the variance of the least squares slope estimator without a constant is

$$var(\tilde{b}) = \frac{\sigma^2}{\sum_{i=1}^{n} X_i^2}.$$

   The ratio of the two is

$$\frac{var(\tilde{b})}{var(b)} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sum_{i=1}^{n} X_i^2} = \frac{\sum_{i=1}^{n} X_i^2 - n\bar{X}^2}{\sum_{i=1}^{n} X_i^2} < 1.$$

   Therefore, running a regression with an unnecessary constant inflates the variance of the slope estimator.

2. Greene 5th edition, Exercise 4.4.

   Answer: Rewrite the regression model as

$$\begin{aligned} y_i &= (\alpha + \lambda) + \beta x_i + (\epsilon_i - \lambda) \\ &= \alpha^* + \beta x_i + \epsilon_i^*. \end{aligned}$$

   This model satisfies all our Classical assumptions so that we will get unbiased estimators of α^* and β. Of course, this means that we get a biased estimate of α.

3. Greene 5th edition, Exercise 4.11.

   Answer: See the attached gauss code.

   (a) At least on the surface, the sign on the price of new cars seems counterintuitive. I have no priors on the signs for Pd, Pn and Ps.

   (b) The 5% critical value for $df = 36 - 10 = 26$ equals 2.056. The t statistic is 0.78. Therefore, we fail to reject the null that consumers do not differentiate between changes in the prices of new and used cars.

   (d) The elasticities are comparable, at least in sign. It is difficult to determine which specification is most preferable. We will discuss functional forms later in the course.

(e) If the price indices were simply rescaled via multiplication by constants, then the fit of the model would not chage but the coefficient estimates would change accordingly.

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@ ************* @
@ Exercise 4.11 @
@ ************* @

@ Load Data and Create Data Matrices @
load data[37,11] = c:\gauss35\classes\econ5340\data\F2.2;
gas = data[2:37,2];
popul = data[2:37,11];
y = gas./popul;
x = data[2:37,3:10];
nobs = rows(x);
constant = ones(nobs,1);
trend = seqa(1,1,nobs);
xmat = constant~trend~x;
k = cols(xmat);

@ Part (a) @
b = inv(xmat'*xmat)*(xmat'*y);
resids = y - xmat*b;
ssquare = (resids'*resids)/(nobs-k);
varb = ssquare*inv(xmat'*xmat);
demeany = y - meanc(y);
rsquare = 1 -  (resids'*resids)/(demeany'*demeany);
print "b vector = " b;
print;
print "Rsquare = " Rsquare;
print;


b vector =
   -0.0092877873
    0.0094023473
     -0.12067711
   0.00011136385
     0.063498315
    -0.040848538
     0.058881236
      0.28961515
      0.54240186
     -0.87698183

Rsquare =       0.98294092

@ Part (b) @
tstat = (b[5,1] - b[6,1])/sqrt(varb[5,5] + varb[6,6] - 2*varb[5,6]);
print "t stat = " tstat;
print;

t stat =       0.78100209

@ Part (c) @
incavg = meanc(xmat[.,4]);
gasavg = meanc(xmat[.,3]);
pubavg = meanc(xmat[.,7]);
yavg = meanc(y);
gaselas = b[3,1]*(gasavg/yavg);
incelas = b[4,1]*(incavg/yavg);
pubelas = b[7,1]*(pubavg/yavg);
print "Own price elasticity = " gaselas;
print "Income elasticity = " incelas;
print "Cross price elasticity = " pubelas;
print;
```

```
Own price elasticity =      -0.27764522
Income elasticity =        1.0211574
Cross price elasticity =       0.16051273

@ Part (d) @
tempx = xmat[.,3:10];
tempx2 = ln(tempx);
lnxmat = constant~trend~tempx2;
lny = ln(y);
elas = inv(lnxmat'*lnxmat)*(lnxmat'*lny);
print "elasticities = " elas;

elasticities =
      -10.422716
   -0.0043973175
     -0.53482757
       1.2151427
     0.083131298
     -0.11498662
      0.12063180
      0.94285095
       1.2174980
      -1.3066733
```