

# ECON 5350 Problem Set #1

Due: Tuesday, September 15 at the beginning of class

1. Suppose that a lottery ticket costs \$1 per play. The game is played by drawing six numbers (order matters) without replacement from the numbers 1 to 48. If you guess all six numbers, you win the prize (assume only one player wins). Now suppose that  $N$  equals the number of tickets sold and  $P$  equals the size of the prize.  $N$  and  $P$  are related by

$$N = 5 + 1.2P$$

$$P = 1 + 0.4N$$

$N$  and  $P$  are measured in millions. What is the expected value of a ticket in this game?

2. Chebyshev's Inequality. For the following two probability distributions, find the lower limit of the probability of the indicated event using Chebyshev's inequality and the exact probability using the appropriate table.

(a)  $X \sim N(0, 3^2)$  and  $-4 < X < 4$ .

(b)  $X \sim \chi^2(8)$  and  $0 < X < 16$ .

3. Use the following joint probability distribution

		$X$		
		0	1	2
	0	0.05	0.1	0.03
Y	1	0.21	0.11	0.19
	2	0.08	0.15	0.08

to complete the following

- (a) Compute  $\Pr(Y < 2)$ ,  $\Pr(Y < 2, X > 0)$  and  $\Pr(Y = 1, X \geq 1)$ .
  - (b) Find the marginal distributions of  $X$  and  $Y$ .
  - (c) Calculate  $E(X)$ ,  $E(Y)$ ,  $Var(X)$ ,  $Var(Y)$ ,  $Cov(X, Y)$  and  $E(X^2Y^3)$ .
  - (d) Calculate  $Cov(Y, X^2)$ .
  - (e) What are the conditional distributions of  $Y$  given  $X = 2$  and of  $X$  given  $Y > 0$ ?
  - (f) Find  $E(Y|X)$  and  $Var(Y|X)$ .
4. Find the MGF of the exponential and Poisson distributions.

5. Use the MGFs to find the mean and variance of the exponential and Poisson distributions.

### **GAUSS PROBLEMS**

6. Generate a  $(n \times 1)$  vector of independent draws from a  $N(0, 1)$  distribution where  $n = 100$ . Calculate the sample mean ( $\bar{X}$ ), sample variance( $s^2$ ) and a histogram of the sample pdf.
7. Repeat Exercise 6 using a  $\chi^2(1)$  distribution.