ECON 5350 Midterm Exam – Spring 2007

1. **Generalized Least Squares (25 pts).** Consider the following partitioned regression model:

\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} =
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} \beta +
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2
\end{bmatrix} 
\]  

(1)

where

\[
\Omega =
\begin{bmatrix}
\sigma^2 \mathbf{I}_{n_1} & 0 \\
0 & \sigma^2 \mathbf{I}_{n_2}
\end{bmatrix}
\]

is the variance covariance matrix of the error terms. Find the formula for the GLS estimator and suggest a feasible GLS estimator.

2. **Autocorrelation (30 pts).** Consider the simple linear regression model \( y_t = \beta_0 + \beta_1 x_t + \epsilon_t \), where the errors exhibit second-order autocorrelation: \( \epsilon_t = \rho_2 \epsilon_{t-2} + \mu_t \).

(a) Calculate the autocovariance function (i.e., \( \gamma(s) = \text{cov}(\epsilon_t, \epsilon_{t-s}) \)) under the assumption that \( \epsilon_t \) is a weakly covariance stationary process.

(b) Assuming \( \rho_2 \) is known, derive the GLS estimator of \( \beta_1 \).

(c) Describe how one would calculate an asymptotically efficient two-step estimator of \( \beta_1 \) if \( \rho_2 \) were unknown.
3. SUR and Panel-Data Models (45 pts). Consider the following SUR system:

\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} =
\begin{bmatrix}
X_1 & 0 \\
0 & X_2
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} +
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2
\end{bmatrix}
\]  \tag{2}

where \(Y_1, Y_2, \epsilon_1\) and \(\epsilon_2\) are of dimension \((T \times 1)\); \(X_j\) is of dimension \((T \times k_j)\) for \(j = 1, 2\); \(\beta_j\) is of dimension \((k_j \times 1)\) for \(j = 1, 2\); and the system variance-covariance matrix is

\[
\Omega = \Sigma \otimes I_T = \begin{bmatrix}
\sigma_1^2 I_T & \sigma_{12} I_T \\
\sigma_{12} I_T & \sigma_2^2 I_T
\end{bmatrix}
\]

where

\[
\Omega^{-1} = \Sigma^{-1} \otimes I_T = \begin{bmatrix}
a_{11} I_T & a_{12} I_T \\
a_{12} I_T & a_{22} I_T
\end{bmatrix}.
\]

(a) The SUR system above could be used to incorporate panel data. Describe an estimation strategy to obtain consistent, asymptotically efficient estimates for a fixed effects version of Model (2).

(b) Show that equation-by-equation OLS is equivalent to GLS when \(X_1 = X_2\). For simplicity, consider the case when \(k_1 = k_2 = 1\) and \(T = 2\).

(c) Describe, in detail, how to test Model (1) versus Model (2) when \(\sigma_{12} = 0\). Continue to assume that \(X_1 = X_2, k_1 = k_2 = 1\) and \(T = 2\).