Panel Data (100 pts). Consider the following two-way panel data model

\[ y_{i,t} = \alpha_i + x_{i,t}'\beta + \gamma_t + \epsilon_{i,t} \]

where \( i = 1, \ldots, n \), \( t = 1, \ldots, T \) and one of the following four assumptions hold:

1. \( \alpha_i \) and \( \gamma_t \) are unknown parameters and \( \epsilon_{i,t} \) is a mean-zero independent random variable with variance \( \sigma_{\epsilon,t}^2 \).
2. \( \alpha_i \) and \( \gamma_t \) are unknown parameters and \( \epsilon_{i,t} = \rho \epsilon_{i,t-1} + \nu_{i,t} \), where \( \nu_{i,t} \sim i.i.d. \( (0, \sigma_{\nu}^2) \). \)
3. \( \gamma_t \) is an unknown parameter, \( \alpha_i \) is a mean-zero independent random variable with variance \( \sigma_{\alpha,i}^2 \) and \( \epsilon_{i,t} \sim i.i.d. \( (0, \sigma_{\epsilon}^2) \). \)
4. \( \alpha_i \) is an unknown parameter, \( \gamma_t \) is a mean-zero random variable, \( \gamma_t = \rho \gamma_{t-1} + \nu_t, \nu_t \sim i.i.d. \( (0, \sigma_{\nu}^2) \) \) and \( \epsilon_{i,t} \sim i.i.d. \( (0, \sigma_{\epsilon}^2) \). \)

Random variables \( \alpha, \gamma \) and \( \epsilon \) are mutually independent. For each of the four cases above, write out the full variance-covariance matrix of the errors when \( n = 2 \) and \( T = 3 \). For cases 1 and 2, outline an estimation strategy that will produce consistent and asymptotically efficient estimates of \( \beta \) when \( n \) and \( T \) are large.