1. **Systems of Equations (50 pts).** Consider the following structural simultaneous equation model:

\[
\begin{align*}
y_{1,t} &= \beta_x x_t + \gamma_1 y_{2,t} + \epsilon_{1,t} \\
y_{2,t} &= \beta_w w_t + \beta_z z_t + \gamma_2 y_{1,t} + \epsilon_{2,t}
\end{align*}
\]

where all variables are in deviation-from-mean form, the vector \( \{x, w, z\} \) is exogenous, and \( t = 1, \ldots, T \).

(a) Find the reduced-form equations.

(b) Propose an estimation strategy for the reduced-form parameters. How does your estimation strategy compare to estimation of an SUR model? Explain.

(c) Are the structural parameters identified? Explain.

(d) Formally show that OLS estimates of \( \gamma_1 \) are biased.

(e) Describe how to perform GMM estimation of \( \beta_x \) and \( \gamma_1 \).

2. **Panel Data (50 pts).** Consider a simple panel model with one exogenous variable, \( x_{i,t} \).

(a) Write down a two-way fixed effects (FE) model, clearly defining all variables and the notation.

(b) Write down the standard random effects (RE) model and the associated variance-covariance matrix of the errors, \( \Omega \).

(c) Form the matrix equivalent of your model in part (a).

(d) What is the variance-covariance matrix for the OLS estimator, \( b \), of the RE model? What are the tradeoffs between \( b \) and \( \hat{\beta}_{RE} \)?

(e) Develop a Hausman test for FE vs. RE. Write a short paragraph that includes some intuition for how the test works.

3. **Probit Model (50 pts).** Consider a probit model for passing this class (i.e., receiving an 80% or better):

\[
y_i^* = \beta_0 + x_i \beta_1 + \epsilon_i,
\]

where \( y_i^* \) is your latent cumulative score and \( x \) is the number of hours devoted to studying per week.

(a) How would you estimate \( \beta_0 \) and \( \beta_1 \) if you had data on cumulative scores? Could you identify the parameters separately from \( \sigma \)?

(b) Now assume you only have data on whether the student passes (\( y_i = 1 \)) or fails (\( y_i = 0 \)). Carefully form the likelihood function.

(c) Find the first-order conditions (FOCs) for the probit model.

(d) Propose a strategy to solve the FOCs and complete the estimation.

(e) Find an expression for the marginal effect, \( \delta_x \), of one more hour studying on the probability of passing this class. Show how to find the associated standard error of \( \hat{\delta}_x \).