Pollution Permits, Green Taxes, and the Environmental Poverty Trap

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February 13, 2017

Abstract

We compare pollution permits and green taxes in a unified overlapping generations model with endogenous longevity. The model identifies pollution permits as a potential source of multiple equilibria. One nontrivial equilibrium is an environmental poverty trap (EPT) with low capital and a high stock of pollution. An economy operating around the equilibrium will gravitate toward this equilibrium in the long run. The other nontrivial equilibrium is a desirable one with high capital and a low stock of pollution. A saddle path leads to this desirable equilibrium. Alternatively, green taxes produce a unique stable equilibrium that avoids the EPT. Our conclusion is that developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions but proceed with caution given the possibility of being drawn into an EPT.

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1. Introduction

Developing countries face pressing economic and environmental challenges. Economic growth in developing countries usually comes with severe environmental degradation in terms of worsening air, water, and soil quality. According to a report by the World Bank Group, the readings of air pollution indicators are higher in low-and-middle-income countries than in high-income countries (World Bank Group, 2015a). The health effect of pollution is staggering. Environmental degradation is estimated to be responsible for about 40% of world deaths (Pimentel et al., 2007). Air pollution alone is estimated to cause 3.3 million premature deaths each year around the world, notably in China and India (Lelieveld, Evans, Fnais, Giannadaki, & Pozzer, 2015). To address environmental deterioration, developing countries can adopt environmental regulations such as pollution permits and green taxes. But can the cure make the situation worse if it is not properly implemented? In this paper, we address that question by comparing the dynamics of the economy and the environment in a developing country that is considering either pollution permits or green taxes to manage the environment. Our analysis shows that pollution permits, if not properly managed, may endogenously give rise to an environmental poverty trap (EPT) whereas green taxes do not. An EPT refers to an equilibrium outcome with low capital and high pollution, and emerges in dynamic economic and environmental systems with multiple equilibria. The idea of EPT is not new, but this paper identifies a new mechanism that contributes to the emergence of EPT. EPTs have been shown to be generated by threshold effects of environmental quality on longevity or survival probability (Mariani, Pérez-Barahona, & Raffin, 2010), endogenous technical choice in the presence of emission taxes (Varvarigos, 2014), nonlinear regeneration of environmental quality (Prieur, Jean-Marie, & Tidball, 2013; Dao & Edenhofer, 2014), and public pollution abatement financed
by public debt (Fodha & Seegmuller, 2014). The emergence of EPT in this paper, however, does not depend on the mechanisms identified by the extant literature, but instead depends only on usual assumptions in terms of technology, preference, and the law of motion for the stock of pollution in the context of pollution permits. The primary contribution of this paper thus enriches the literature on EPT by providing a new mechanism that may potentially lead to an EPT.

The possibility of an EPT under pollution permits is revealed by the marriage of two modeling frameworks, Stokey (1998) and Chakraborty (2004). These two frameworks are used by a wide spectrum of theoretical literature that unifies the environment with economic growth. Pollution is modeled as associated with either production or consumption. In line with this modeling approach, the Stokey (1998) framework allows for mathematical transformation of pollution as being a byproduct of final output into a necessary input in production. By tailoring the interpretation of the price of pollution in production within the Stokey (1998) framework, we flexibly analyze and compare the dynamic systems under pollution permits and under green taxes in a unified modeling framework. Pollution nevertheless comes with consequences on economic growth. The most common modeling approach is that pollution imposes detrimental effect on agents’ utility or on productivity (see Xepapadeas, 2005 for a fine summary). Pollution also can affect economic growth by endogenously altering agents’ time preference (Chu, Lai, & Liao, 2015; Vella, Dioikitopoulos, & Kalyvitis, 2015). In recent years, the literature has specifically shed light on the effects of pollution on economic growth through the health-effect channel. Because pollution makes people sick, the effective labor supply is reduced (Chen, Shieh, & Chang, 2015). Pollution increases morbidity, so agents engage in precautionary savings to pay for medical expenses (Wang, Zhao, & Bhattacharya, 2015). Pollution also affects mortality/longevity/life expectancy, thus modifying agents’ incentive to save (Pautrel, 2009;
Jouvet, Pestieau, & Ponthiere, 2010; Varvarigos, 2010; Raffin & Seegmuller, 2014a, 2014b; Varvarigos, 2014). The common feature of the last strand of literature is that it augments the OLG model with endogenous longevity developed by Chakraborty (2004). In this strand of literature, longevity is affected negatively by pollution, while positively by such factors as private and public healthcare spending, either in absolute terms or in relative terms (healthcare expenditure as a percentage of GDP), and income per capita. By linking longevity, pollution, savings, and capital within the Chakraborty (2004) framework, we establish the connection between the economy and the environment. To the best of our knowledge, we are the first to combine the frameworks of Stokey (1998) and Chakraborty (2004) to compare pollution permits and green taxes.

We also note that our paper is closely related to the seminal article by Weitzman (1974), who compares economic control via prices or via quantities. Weitzman argues that from a strictly theoretical perspective, it is equivalent to regulate an autarky economy by restricting either prices or quantities under certainty. Our paper finds the same equivalence but also highlights a key difference between environmental regulations with prices versus with quantities even in a certain world. Although the same steady-state level of welfare can be achieved with either a properly selected green-tax rate or a pollution quota, the choice of control can have important implications for the existence of multiple equilibria and the associated transition dynamics. Sim and Lin (2016) also point out the difference between quantity control versus price control in a certain world, but they focus on a globalized and static economy rather than on an autarky and dynamic setting employed by this paper. Therefore, another contribution of our paper is to fully consider the transition dynamics, thus augmenting the ‘prices vs. quantities’ debate surrounding the appropriate nature of environmental regulation.
Inspired by the literature on endogenous longevity, we consider a basic two-period OLG model with agents’ longevity improved by income per capita but reduced by the stock of pollution. The stock of pollution accumulates due to economic activity and declines due to public environmental maintenance. If some technical conditions are satisfied, multiple equilibria endogenously arise under pollution permits. One nontrivial equilibrium is an EPT with low capital and high pollution. The economy will be trapped in the EPT unless the government intervenes to steer the economy towards the other nontrivial equilibrium, a desirable one with high capital and low pollution. Alternatively, no EPT exists under green taxes because there is only one stable equilibrium. We also consider an alternative model that further incorporates private healthcare expenditure as another endogenous factor improving agents’ longevity. Our main results qualitatively carry through.

The multiple equilibria under pollution permits can be explained as follows. The law of motion for capital implies a negative relationship between pollution and capital. A high stock of pollution implies a low level of capital since agents save little due to their short longevity. Similarly, a low stock of pollution implies a high level of capital due to high savings. Can these two combinations be supported in the long run under environmental regulation? With pollution permits, the flow of pollution is fixed but a high level of capital implies more funds for environmental maintenance, leading to a low stock of pollution. Conversely, a low level of capital leads to insufficient funds for environmental maintenance so that the stock of pollution is high. Either combination can be supported in the long run. Under green taxes, however, emissions are not fixed but are relatively low when the economy is depressed. This implies a low level of capital and a low stock of pollution. As the economy grows and the level of capital becomes higher, emissions are higher and the stock of pollution is larger. This means that the
only feasible combination of pollution and capital under green taxes lies somewhere between the two extremes, and thus multiple equilibria are not supported.

We wish to highlight three main results. First, a developing economy implementing pollution permits might be pulled into an EPT unless the government encourages agents to increase savings. In contrast, a developing economy using green taxes will always converge to the unique equilibrium that can replicate the desirable equilibrium under pollution permits, such that the EPT can be avoided. Second, a well-designed environmental regulation with pollution permits can generate approximately the same discounted sum of welfare as under green taxes along the transition path. Third, multiple equilibria under pollution permits emerge for a wide range of parameters. The main policy implication of our research is that developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions but proceed with caution given the possibility of being drawn into an EPT.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 describes the equilibria and dynamics when pollution permits are implemented. Section 4 describes the equilibrium and dynamics when green taxes are employed. Section 5 provides numerical simulations to (i) contrast the key variables along the transition paths, (ii) compare the long-run impacts from changes in parameters under pollution permits versus under green taxes, and (iii) investigate how likely the emergence of multiple equilibria is under pollution permits. Section 6 discusses practical implications of our model and concludes.
2. The Basic Setup

2.1 The production function

The production function is specified as

\[ Y_t = B K_t^\alpha L_t^\beta P_t^{1-\alpha-\beta}, \]  

where \( K_t \) is physical capital, \( L_t \) is the aggregate labor supply, \( P_t \) is the flow of pollution, \( B \in (0, +\infty) \) is a production function scalar, and \( \alpha, \beta \in (0,1) \) are parameters with \( 1 - \alpha - \beta > 0 \).

This production function is in line with the common modeling method where the flow of pollution is a byproduct of final output. After some mathematical manipulation of the Stokey (1998) model, the flow of pollution can equivalently be treated as a necessary input in the production function (Ono, 2002; Brock & Taylor, 2005; Jouvet, Michel, & Rotillon, 2005a, 2005b; Prieur et al., 2013). This is a convenient modeling framework for comparing pollution permits and green taxes.

2.2 Firms

The economy is perfectly competitive. In each period, the representative firm faces a static profit-maximization problem subject to the production function (1). Denote \( r_t \) as the rental price of capital, \( w_t \) as wage rate, and \( q_t \) as the “price” of pollution, all of which are exogenous to the firm. The term \( q_t \) can be interpreted either as the price of pollution permits if the government auctions pollution permits to the firm\(^1\) or as the marginal tax rate if the government levies a green tax on the firm. Although \( q_t \) can have two different interpretations, the profit-

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\(^1\) Pollution permits can be distributed to firms either by “grandfathering”, i.e., the government directly gives pollution permits to firms for free, or by auctioning. Jouvet et al. (2005b) show that all pollution permits should be auctioned, because auctioning is the most efficient way of distributing pollution permits. Although our model can be easily adapted to allow for the case of “grandfathering”, we focus on the case where pollution permits are auctioned.
maximization problem for the firm takes the same form. The firm chooses capital $K_t$, labor $L_t$, and the flow of pollution $P_t$ as production inputs to maximize profits $\pi_t$. The problem is

$$\max \pi_t = BK_t^\alpha L_t^\beta P_t^{1-\alpha-\beta} - r_t K_t - w_t L_t - q_t P_t.$$ 

The first-order conditions require the marginal product of each input to equal its price:

$$r_t = \alpha Bk_t^{-1} p_t^{1-\alpha-\beta}, \quad (2)$$

$$w_t = \beta Bk_t^{-1} p_t^{1-\alpha-\beta}, \quad (3)$$

$$q_t = (1-\alpha-\beta) Bk_t^{-1} p_t^{1-\alpha-\beta}, \quad (4)$$

where $k_t = K_t/L_t$ and $p_t = P_t/L_t$ are capital and flow of pollution in per-capita terms. Perfect competition drives the representative firm’s profits to zero.

Consider the case in which the government auctions pollution permits to manage the environment. The government sets a pollution quota per capita, $\bar{p}_t$, and auctions off the pollution permits representing the quota. Equation (4) then determines each firm’s willingness to pay for pollution permits with $q^{mp}(k_t, \bar{p}_t)$ equal to the equilibrium auction price of pollution permits.\(^2\) The representative firm treats $\bar{p}_t$ as exogenous and $q^{mp}(k_t, \bar{p}_t)$ is set endogenously through the auctioning of pollution permits. The equilibrium price of pollution permits increases in capital and decreases in the amount of pollution allowed by the permits.

\(^2\) In equilibrium, the amount of pollution that firms choose as a production input, $p_t$, is equal to the amount of pollution allowed by pollution permits, $\bar{p}_t$. We assume that the government consistently cuts pollution, implying that $\bar{p}_t$ is always a binding constraint.
Also consider the case in which the government levies green taxes to manage the environment. In this case, \( q_i \) is interpreted as the tax rate per unit of pollution emitted rather than as the auction price of pollution permits. With green taxes, the firm treats tax rate \( q_i \) as an exogenous variable set by the government and the firm chooses the optimal level of emissions, \( p^{gr}(k_i, q_i) \), determined by (4). The flow of pollution emitted increases in capital and decreases in the green-tax rate.

### 2.3 Government

The government maintains a balanced budget in each period. We assume that each period’s fiscal revenues from controlling pollution, \( q_iP_t \), serve as the sole source of income for the government. With these fiscal revenues, the government finances public environmental maintenance \( A_t \) that prevents pollution from growing.\(^3\) Denote \( a_t = A_t/L_t \) as per-capita public environmental maintenance. If the government implements environmental regulation with pollution permits, the balanced budget in per-capita terms is

\[
a_t^{pp} = q^{pp}(k_i, \overline{p}_i)\overline{p}_i = (1 - \alpha - \beta)Bk_i\overline{p}_i^{\alpha - \beta}. \tag{5}
\]

Alternatively, if the government uses green taxes, the balanced budget in per-capita terms is

\[
a_t^{gr} = q_i p^{gr}(k_i, q_i) = (1 - \alpha - \beta)\left(\frac{1 - \alpha - \beta}{q_i}\right)^{\alpha - \beta} B^{\alpha - \beta} k_i^{\alpha - \beta}. \tag{6}
\]

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\(^3\) Examples of environmental maintenance include building water treatment plants to reduce waste water, cloud seeding and deploying water cannons to reduce urban PM2.5 and PM10 density, as well as planting trees to reduce atmospheric carbon dioxide level.
2.4 Pollution

The stock of pollution $Z_t$ accumulates due to the flow of pollution $P_t$ and declines due to environmental maintenance $A_t$. The stock of pollution evolves according to

$$Z_{t+1} = (1-\theta)Z_t - \gamma A_t + P_t,$$

where $\theta \in (0,1]$ measures the speed of autonomous dissipation of the stock of pollution, and $\gamma > 0$ is a parameter representing the efficiency of environmental maintenance. Dividing equation (7) through by $L_t$ and assuming no population growth yields the evolution of the per-capita stock of pollution

$$z_{t+1} = (1-\theta)z_t - \gamma a_t + p_t.$$

2.5 Agents

A representative agent lives two periods, which is divided into “young” and “elderly” categories. An agent lives the entirety of her young period but only lives the fraction $\phi$ of her elderly period. An agent born at the beginning of period $t$ therefore has lifetime longevity equal to $1 + \phi_{t+1}$. Assume that $\phi_{t+1}$ depends on the agent’s health status in her young period, $x_t$. An agent’s health status improves in income per capita but deteriorates due to the stock of pollution. Similar to Varvarigos (2014), we treat the health status $x_t$ as the ratio of income per capita to the

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4 The term $\phi$ can be interpreted as either the fraction of the elderly period that an agent lives or the survival probability at the beginning of the elderly period. Interpreting $\phi$ as survival probability does not affect the predictions of the model, but it can raise bequest issues. Because all agents save in the young period, but a fraction of the agents will die at the beginning of the elderly period, the savings of those who die will become income for those who are alive in the elderly period. Bequeathing thus leads the return rate of savings to be larger than the rental rate of capital. In contrast, if $\phi$ is interpreted as deterministic longevity, the return rate of savings will be equal to the rental rate of capital.
adjusted stock of pollution, i.e.,  \( x_t = y_t / \eta z_t \), where \( y_t = Y_t / L_t \) and \( \eta > 0 \) captures the detrimental effect of pollution on the health status.\(^5\) Assume

\[
\phi_{t+1} = \phi(x_t) = \frac{\lambda + \bar{\lambda} x_t}{1 + x_t},
\]

(9)

where the parameters \( \lambda \) and \( \bar{\lambda} \) are the lower and upper bounds of a representative agent’s potential longevity in the elderly period, i.e., \( 0 \leq \lambda \leq \phi_{t+1} < \bar{\lambda} \leq 1 \). Substituting \( x_t = y_t / \eta z_t \) into (9) gives an agent’s longevity as a function of income per capita \( y_t \) and the stock of pollution \( z_t \):

\[
\phi_{t+1} = \phi(y_t, z_t) = \frac{\lambda \eta z_t + \bar{\lambda} y_t}{\eta z_t + y_t}.
\]

(10)

Note that the representative agent takes both income per capita and the stock of pollution as given.

The representative agent born at the beginning of period \( t \) derives utility from consumption in her youth, \( c_t \), and elderly consumption, \( d_{t+1} \). Longevity is taken as given by the agent, but all else equal, she will derive more utility from consumption if she lives longer in her elderly period. To permit closed-form solutions, we assume that the representative agent’s lifetime utility takes a logarithmic form that is additively separable (see, for example, Chakraborty, 2004; Fodha & Seegmuller, 2014; Varvarigos, 2014):

\(^5\) In Appendix C, we consider an alternative model to check the robustness of the results. In this alternative model, the agent’s health status is determined by private healthcare expenditure, income per capita, and the stock of pollution. Although agents still treat income per capita and the stock of pollution as given, they can actively improve their longevity through private healthcare expenditure. We show that the main results derived from the basic model remain robust to this alternative specification.
\[ U_t = \ln c_t + \phi_{t+1} \ln d_{t+1}. \]  

(11)

When the representative agent is young, she inelastically supplies one unit of labor to earn the exogenous wage rate \( w_t \). The agent’s labor income is divided between young consumption, \( c_t \), and savings, \( s_t \). Savings can only be rented to firms in the form of physical capital at the rental price of \( r_{t+1} \). The remunerated savings, \( r_{t+1}s_t \), are returned to the agent at the beginning of her elderly period and are used to finance her elderly consumption, \( d_{t+1} \).

The representative agent faces two budget constraints – one for her young period and one for her elderly period:

\[ w_t = c_t + s_t, \]  

(12)

\[ r_{t+1}s_t = d_{t+1}. \]  

(13)

Solving the agent’s utility-maximization problem gives the saving function

\[ s_t = \Phi_{t+1} w_t = \frac{\phi_{t+1}}{\phi_{t+1} + 1} w_t, \]  

(14)

where \( \Phi_{t+1} = \phi_{t+1}/(\phi_{t+1} + 1) \in \left[ \frac{1}{2}, \frac{1}{\lambda_{t+1}} \right] \) is the agent’s propensity to save. Because the agent’s propensity to save increases in longevity, savings increase in longevity if other things are equal. By (12) and (13), the agent’s young and elderly optimal levels of consumption are

\[ c_t = \frac{1}{\phi_{t+1} + 1} w_t, \]  

(15)
\[ d_{t+1} = r_{t+1} \frac{\phi_{t+1}}{\phi_{t+1} + 1} \]  

(16)

Other things equal, equation (15) indicates that the longer the agent lives, the lower consumption will be in her youth as the agent has to save more to satisfy her elderly consumption. Conversely, equation (16) suggests that the longer the agent lives, all else equal, the higher her elderly consumption will be.

3. Environmental Regulation with Pollution Permits

We now investigate the dynamics of the economy and the environment under environmental regulation with pollution permits. For simplicity, we assume that physical capital completely depreciates within one period, and hereafter treat physical capital as a flow rather than as a stock.

Since agents’ savings in period \( t \) become physical capital in period \( t + 1 \), we have

\[ k_{t+1} = s_t. \]  

(17)

Substituting equation (14) into (17) gives

\[ k_{t+1} = \Phi(y_t, z_t)w_t, \]  

(18)

where \( \Phi_{t+1} = \Phi(y_t, z_t) \) is the agent’s propensity to save as a function of income per capita and the stock of pollution. Consider constant per-capita pollution permits, \( \bar{p}_t = \bar{p} \) for all \( t \).

Substituting expressions into (18) under environmental regulation with pollution permits yields the nonlinear difference equation for physical capital:

\[ k_{t+1} = \frac{\lambda \eta z_t + \lambda B \bar{p}^{-\alpha - \beta} k_t^\alpha}{(\lambda + 1)\eta z_t + (\lambda + 1)B \bar{p}^{-\alpha - \beta} k_t^\alpha} \beta B \bar{p}^{-\alpha - \beta} k_t^\alpha. \]  

(19)
From equation (19), we define \( k_{t+1} - k_t = 0 \) as the \( kk^{pp} \) locus under pollution permits:

\[
\text{the } kk^{pp} \text{ locus: } \frac{\lambda \eta z_t + \overline{\lambda} Bp^{1-\alpha-\beta}k_i^\alpha}{(\overline{\lambda} + 1)\eta z_t + (\overline{\lambda} + 1)Bp^{1-\alpha-\beta}k_i^\alpha} \beta Bp^{1-\alpha-\beta}k_i^\alpha - k_t = 0 .
\]

(20)

Similarly, we have a difference equation for the stock of pollution

\[
z_{t+1} = (1-\theta)z_t - \gamma a^{pp}(k_t, \overline{p}) + \overline{p} ,
\]

(21)

which after substitutions gives

\[
z_{t+1} = (1-\theta)z_t - \gamma (1-\alpha-\beta)Bp^{1-\alpha-\beta}k_i^\alpha + \overline{p} .
\]

(22)

Finally, from equation (22), we define \( z_{t+1} - z_t = 0 \) as the \( zz^{pp} \) locus under pollution permits:

\[
\text{the } zz^{pp} \text{ locus: } -\theta z_t - \gamma (1-\alpha-\beta)Bp^{1-\alpha-\beta}k_i^\alpha + \overline{p} = 0 .
\]

(23)

For \( k_0, z_0 > 0 \), the dynamics of the economy and the environment under environmental regulation with pollution permits are determined jointly by difference equations (19) and (22).

In our subsequent analysis, we make use of the following

Assumptions: (i) \( 0 < \alpha < \beta < 1 \), and (ii) \( \underline{\lambda} = 0 \) and \( \overline{\lambda} = 1 \).

Assumption (i) says capital’s share in production is smaller than labor’s share, which is consistent with the empirical evidence (see, for example, Mankiw, Romer, & Weil, 1992; Gollin, 2002). This assumption, combined with the condition \( 1-\alpha-\beta > 0 \), implies that \( 0 < \alpha < 1/2 \).

Assumption (ii) will greatly simplify the math but does not qualitatively alter the results.
3.1 Steady States

We summarize the necessary conditions for the emergence of multiple equilibria in proposition 1. Then we focus on the interesting case where multiple equilibria emerge.

Proposition 1. When \( \gamma \leq \frac{2}{1-\alpha-\beta} \frac{\alpha}{\eta} \), a unique equilibrium exists; when \( \gamma > \frac{2}{1-\alpha-\beta} \frac{\alpha}{\eta} \), and

\[
\frac{1}{\eta} \beta B^2 \bar{p} 2^{(1-\alpha-\beta)} < T(k^*),
\]

where

\[
T(k) = \frac{k}{\eta} k^{1-2\alpha} + \left[ \frac{2}{\eta} - \frac{\gamma}{\beta} (1-\alpha-\beta) \right] B p^{1-\alpha-\beta} k^{1-\alpha} \quad \text{and}
\]

\[
k^* = \left[ \frac{1-2\alpha}{1-\alpha} \frac{p^{*\alpha}}{\eta (1-\alpha-\beta) - 2\alpha \beta} \right]^\frac{1}{2},
\]

the emergence of multiple (dual) equilibria is possible.

Proof. See Appendix A, Proof #1.

The proposition says that for the environmental maintenance efficiency \( \gamma \), there exists a threshold value \( \frac{2}{1-\alpha-\beta} \frac{\alpha}{\eta} \), which consists of a scalar \( \frac{2}{1-\alpha-\beta} \) determined by the upper bound of longevity and pollution’s share in production, and the dissipation rate for the stock of pollution adjusted by the detrimental effective of pollution on longevity \( \eta \). If \( \gamma \) does not surpass this threshold value, the pollutant is relatively difficult to clean up and only one nontrivial steady state exists. However, if \( \gamma \) is larger than this threshold value, the pollutant is easier to clean up and the emergence of multiple equilibria is possible.

Figure 1 illustrates the capital-pollution dynamics with multiple equilibria under pollution permits. The \( kk^{pp} \) locus defines all combinations of \( k_i \) and \( z_i \) where capital is in steady state.

The \( kk^{pp} \) locus slopes down because, all else equal, a lower stock of pollution leads to increased longevity, more savings, and a higher level of capital. The \( zz^{pp} \) locus defines all combinations of \( k_i \) and \( z_i \) where the stock of pollution is in steady state. The \( zz^{pp} \) locus slopes down because
economic growth drives up the demand for pollution permits. This causes the price of pollution permits to increase and raises public environmental maintenance, which in turn reduces the stock of pollution. The analytical proofs of the slopes of the \( kk^{pp} \) locus and the \( zz^{pp} \) locus are shown in Appendix A. There are three steady states – two nontrivial steady states and \( k = z = 0 \).\(^6\) The two nontrivial steady states are labeled \((k^l, z^h)\) and \((k^h, z^l)\), where the superscripts denote “low” and “high”. The former steady state leads to reduced longevity, low young and elderly consumptions, low welfare, and thus should be avoided. In contrast, the latter steady state is associated with improved longevity, high young and elderly consumptions, high welfare, and thus is desirable.

What is the intuition behind the existence of two steady states? The key is that environmental maintenance increases in capital for a given value of \( \bar{p} \) (see equation (5)). When the level of capital is low, the environment deteriorates because the discrepancy is relatively large between the flow of pollution allowed by \( \bar{p} \) and the effective environmental maintenance. In the undesirable equilibrium, agents’ longevity is reduced due to the deteriorated environment and they save less, thus reinforcing the fact that the level of capital is low and the stock of pollution is high. When the level of capital is high, however, the environment improves because the discrepancy is relatively small between the flow of pollution allowed by \( \bar{p} \) and the effective environmental maintenance. In the desirable equilibrium, agents’ longevity is improved due to the ameliorated environment and they save more, thus reinforcing the fact that the level of capital is high and the stock of pollution is low.

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\(^6\) Point A in Figure 1 is a trivial steady state where the economy shuts down. Capital declines to zero because the stock of pollution converges to positive infinity such that agents’ longevity is zero. Individuals do not save for consumption in the elderly period of life, and no capital is generated. Because capital is a necessary input in production, the economy shuts down and no pollution will be emitted. The human impact on the evolution of stock of pollution is gone and the stock of pollution gradually goes back to its natural equilibrium, i.e., the stock of pollution is zero.
Consider the following example to highlight the intuition behind the EPT. Suppose a developing economy is poor, the government revenues can only support the building of a few water treatment plants. As there is a large gap between the waste water discharged allowed by the number of pollution permits and the total capacity of water treatment plants, the water quality will be poor, which in turn reduces agents’ longevity and savings. Since savings are low, the economy will be depressed with a low level of capital, which will not generate enough resources to finance the building of any additional water treatment plants. The cycle continues and the country is stuck in an EPT. Alternatively, if the government can encourage sufficient savings, more water treatment plants can be built. The gap between waste water discharged and the capacity of water treatment plants is small, so an ameliorated water quality will lead to increased longevity, savings, and welfare. The EPT is avoided, but it may require government intervention to push the economy onto the desirable transition path. We now turn to a more rigorous examination of the dynamic properties around the steady states under pollution permits.

3.2 Transition Dynamics

We summarize the transition dynamics under pollution permits in the following proposition:

Proposition 2. The steady state \((k^l, z^l)\) is a stable equilibrium. The steady state \((k^h, z^l)\) is saddle if \(z^l > \bar{p}/\left(\frac{1+\alpha}{\alpha} - \frac{\alpha}{E^{pp}} + 2\right)\) and is unstable if \(z^l < \bar{p}/\left(\frac{1+\alpha}{\alpha} - \frac{\alpha}{E^{pp}} + 2\right)\), where \(E^{pp}\) is the elasticity of the propensity to save with respect to income per capita evaluated at the steady state \((k^h, z^l)\).

Proof. See Appendix A, Proof #4.

We are interested in the case where the desirable steady state can be reached and exhibits saddle stability, and draw directional arrows around the steady states in Figure 1. The directional
arrows show that point B is a locally stable EPT. For an initial stock of pollution in the vicinity of point B, there is a continuum of capital levels (savings choice in the previous period) that will cause the system to gravitate toward point B. This is an undesirable equilibrium path that results in a relatively low level of capital and a high stock of pollution. In contrast, the directional arrows around point C (i.e., the desirable steady state) show a saddle path with a unique level of capital (savings) that allows the economy to converge to the desirable steady state with a high level of capital and a low stock of pollution.

4. Environmental Regulation with Green Taxes

We now consider the equilibrium properties of the economy and the environment in which green taxes are imposed to regulate the environment. Applying equation (18) with a constant tax rate $q_t = q$ on the flow of pollution gives the difference equation for physical capital

$$k_{t+1} = \frac{\lambda \eta z_t + \lambda \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho} \beta \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho}}{(\lambda + 1) \eta z_t + (\lambda + 1) \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho}}. \quad (24)$$

Using equation (24), we define $k_{t+1} - k_t = 0$ as the $kk^{\pi}$ locus under green taxes:

$$\text{the } kk^{\pi} \text{ locus: } \frac{\lambda \eta z_t + \lambda \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho} \beta \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho}}{(\lambda + 1) \eta z_t + (\lambda + 1) \left( \frac{1 - \alpha - \beta}{q} \right) \frac{1 - \alpha - \beta}{\alpha + \beta} B^{\pi + \rho} k_i^{\pi + \rho}} - k_t = 0. \quad (25)$$

The law of motion for the stock of pollution is

$$z_{t+1} = (1 - \theta) z_t + (1 - \gamma q) \left( \frac{1 - \alpha - \beta}{q} \right)^{\pi + \rho} B^{\pi + \rho} k_i^{\pi + \rho}. \quad (26)$$
From equation (26), we define $z_{i+1} - z_i = 0$ as the $zz^g$ locus under green taxes:

\[
-\theta z_i + (1 - \gamma q) B^{\frac{1}{\gamma}} k_i^{\frac{1}{\gamma}} = 0.
\]

For $k_0, z_0 > 0$, equations (24) and (26) jointly determine the capital-pollution dynamics in an economy with green taxes being implemented.

### 4.1 Steady State

In this section we examine the steady state for the case where green taxes are implemented. Figure 2 shows both the $kk^g$ locus with all the combinations of $k_i$ and $z_i$ where capital is in steady state, and the $zz^g$ locus with all the combinations of $k_i$ and $z_i$ where the stock of pollution is in steady state. Similar to Figure 1, the $kk^g$ locus slopes down because less pollution leads to increased longevity, more savings, and a higher level of capital. However, the $zz^g$ locus slopes up. The $zz^g$ locus slopes up because unlike under pollution permits, the flow of pollution is not fixed under green taxes but continues to rise with economic growth. The fact that the $kk^g$ locus slopes down and the $zz^g$ locus slopes up implies that there is only one nontrivial steady state and no EPT under a green-tax system. This is one of our primary findings. The proofs that the $kk^g$ locus slopes down and the $zz^g$ locus slopes up are shown in Appendix B.

---

7 In order to examine the non-trivial solution area, we focus on the case of interest where $1 - \gamma q > 0$. This case implies that the flow of pollution is greater than the effective environmental maintenance in each period, i.e., $p^g(k_i, q) - \gamma q p^g(k_i, q) > 0$. First, the case makes intuitive sense in that a pollutant that is hard to eliminate (represented by a low value for $\gamma$) justifies a high green-tax rate $q$ to control the pollutant from its source. Second, the case is not an uncommon assumption in the literature. Economides and Philippopoulos (2008) argue that it is “too good to be true” that the effective cleanup activity is greater than the polluting effect of production activity. Raffin and Seegmuller (2014a) argue that this assumption “enforces an additional upper bound only on the environmental tax rate, and consequently on the level of preventive expenses.” Third, this assumption is inherently consistent with the law of motion for the stock of pollution when the government implements pollution permits. By
4.2 Transition Dynamics

We summarize the transition dynamics under green taxes in the following proposition:

**Proposition 3.** If Assumption (i) $0 < \alpha < \beta < 0$ holds, the system always converges to the unique steady state under green taxes. Let $E^{\text{gr}}$ be the elasticity of propensity to save with respect to income per capita evaluated at the unique steady state under green taxes. When $E^{\text{gr}}$ satisfies
\[
\left[\frac{\alpha}{\alpha+\beta} (E^{\text{gr}} + 1) - (1 - \theta)\right]^2 - 4\theta \frac{\alpha}{\alpha+\beta} E^{\text{gr}} < 0,
\]
the eigenvalues are complex conjugates and the convergence is cyclical; when $E^{\text{gr}}$ satisfies
\[
\left[\frac{\alpha}{\alpha+\beta} (E^{\text{gr}} + 1) - (1 - \theta)\right]^2 - 4\theta \frac{\alpha}{\alpha+\beta} E^{\text{gr}} > 0,
\]
the eigenvalues are real and distinct and the convergence is non-cyclical.

*Proof.* See Appendix B, Proof #2.

The directional arrows in Figure 2 show that the transition dynamics around the nontrivial steady state (point B) are characterized by cycles provided that the eigenvalues are complex conjugates. For a given initial stock of pollution, any choice for the level of capital (i.e., previous period’s savings) will cause the system to cycle into point B. There are some implications associated with the dampened cycle toward the unique steady state under green taxes. For example, the dampened cycle gives rise to intergenerational inequality/inequity (Seegmüller & Verchère, 2004; Schumacher & Zou, 2015) and periodically negative correlation between longevity and economic growth (Varvarigos, 2013).
5. Numerical Simulations: Pollution Permits versus Green Taxes

In this section, we provide numerical simulations to further illustrate the differences between a system regulated by pollution permits versus a system regulated by green taxes. First, we compare the key variables along the transition paths using a set of benchmark parameters. Second, in the context of multiple equilibria under pollution permits, we compare how changes in the parameters affect the steady states. Third, to complement the analysis of the long-run effects on steady states, we focus on the sensitivity of multiple equilibria under pollution permits to different combinations of parameters.

5.1 Comparison of the Transition Paths

Consider a developing economy with a degraded environment that is attempting to reach a desirable equilibrium with high capital and a low stock of pollution. If the government uses pollution permits, it needs to encourage agents in the initial period to adjust savings, thus affecting future capital and placing the economy on the saddle path. If the government resorts to green taxes, however, it only needs to design a proper green-tax rate on the flow of pollution and the system will converge to the desirable equilibrium. The open question is which environmental regulation – pollution permits or green taxes – generates higher welfare along the transition path. To make the two environmental regulations comparable, we start at a common initial point featuring lower capital and a higher stock of pollution relative to those determined by the desirable equilibrium. We also calibrate the green-tax rate within the model so that the unique equilibrium under green taxes is identical to the desirable equilibrium under pollution permits. We consider two scenarios depending on whether the saddle path is reached under pollution permits – one where the government encourages agents to adjust savings to the level on the
saddle path and a second scenario where there is no government intervention in savings. We apply numerical simulations to compare these two environmental regulations with the calibrated parameters shown in Table 1.

Consider the scenario in which the government implementing pollution permits encourages agents to increase savings in the initial period, such that the level of capital is on the saddle path leading to the desirable equilibrium. To compare, we assume only the young generation lives in the initial period, and further define period welfare as the summation of welfare for the young and old agents living in the same period. We denote period 0 as the initial period, so the comparisons between elderly consumption, longevity, and period welfare begin in period 1. We plot the time paths of key variables under pollution permits versus under green taxes in Figure 3. The notable feature of Figure 3 is that while the starting point and the desirable equilibrium are identical across the two environmental regulations, the environmental regulation with pollution permits gives rise to larger fluctuations along the transition paths than the environmental regulation with green taxes. The oscillations under pollution permits are spurred by the large increase in savings. Because the government encourages agents to increase savings in period 0, there is a large increase in the level of capital from period 0 to period 1 and the young consumption in period 0 is greatly reduced relative to that under green taxes. The increase in savings allows for higher consumption during the elderly phase of the lifecycle and for the next generation of young individuals (period 1). However, the following generation of young agents (period 2) choose to optimally reduce their consumption, save more, and shift...

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8 It is possible that a developing economy may not have the national resources available for savings to ensure the economy can jump onto the saddle path in one period. In this case, the nation would need to increase savings over multiple periods, borrow from other nations, or rely on international aid. In our benchmark model and the subsequent simulations, the economy has sufficient wage income to reach the required level of savings in one period and jump onto the saddle path that leads to the desirable long-run equilibrium.
consumption to the elderly phase of the lifecycle. This occurs because longevity is expected to increase due to the sharp reduction in the stock of pollution from environmental maintenance supported by the jump in capital in period 1. After a sharp increase in welfare in period 3 due to the contribution of a greatly boosted longevity caused by the small stock of pollution in period 2, the back-and-forth movement in consumption and other variables eventually dampens as the economy transitions to the desirable equilibrium. In contrast, the green-tax system operates smoothly along the transition path because the flow of pollution is not fixed by pollution permits but determined by the level of capital, which limits the fluctuations in pollution-driven longevity.

Despite the differences across environmental regulations, the discounted sum of welfare under pollution permits over the 20 periods along the transition path is approximately the same as that under green taxes.9

Figure 4 compares the transition paths under pollution permits versus under green taxes in the scenario where there is no government intervention in savings. While the economy regulated by pollution permits gravitates toward the EPT, the economy under green taxes cycles to the desirable equilibrium with a high level of capital and a low stock of pollution.

Given that environmental regulation with pollution permits generates larger fluctuations in the economy and has the potential to drag a developing economy into an EPT, green taxes might be a safer environmental management choice within our framework. Under green taxes, the economy will always converge to the unique equilibrium with small fluctuations in terms of key economic and environmental variables. This unique equilibrium can replicate the desirable

---

9 The discounted sum of welfare under pollution permits is 19.77, whereas the discounted sum of welfare under green taxes is 19.30. The difference in discounted sum of welfare is equivalent to 29.01% of the welfare generated by the initial young consumption under pollution permits, or to 24.68% of the welfare generated by the initial young consumption under green taxes, indicating the difference is relatively modest.
equilibrium under pollution permits, such that the EPT can be avoided. We note, however, that a carefully constructed environmental regulation with pollution permits that achieves saddle-path stability is capable of generating approximately the same discounted sum of social welfare along the transition path. To achieve this outcome with pollution permits, policymakers must encourage the current generation of young agents to sacrifice consumption and save more. This will ensure a high level of capital in the following period and place the economy on the saddle path leading to the desirable equilibrium.

Another policy concern is whether the EPT persists if the number of pollution permits is allowed to decline over time. Policymakers may wish to gradually issue fewer permits to allow for an ever-improving environment. To explore this issue, suppose the government decreases the number of pollution permits from the benchmark value by 2% each period relative to the previous period. The effects of the decreases in pollution permits, all else equal, are organized in Figure 5. We can see that the EPT still exists even if policymakers continually decrease $\bar{p}$, and it is still necessary for policymakers to intervene in terms of encouraging young agents’ savings.

5.2 Comparison of the Long-Run Impacts from Changes in the Parameters

In this section, we compare the long-run effects of parameter changes under pollution permits versus under green taxes. We focus on parameters related to environmental regulation ($\bar{p}$ under pollution permits and $q$ under green taxes), environmental maintenance efficiency ($\gamma$), the detrimental effect of pollution on the health status ($\eta$), and the dissipation rate of the stock of pollution ($\theta$). The environmental regulation parameters are set by the government, while the latter three depend on the type of pollutant and its subsequent health impact. In Appendix D, we
provide analytical results of how the parameters affect the level of capital and the stock of pollution in the steady state under the two types of environmental regulations.

Under pollution permits, we specifically consider the case where multiple equilibria exist\(^{10}\), and the second column of Table 2 gives the ranges of each parameter that give rise to multiple equilibria while holding other parameters constant. The signs in Table 2 show how the level of capital, the stock of pollution, longevity, and period welfare in the steady state change as the key parameters increase under pollution permits. An increase in \(\bar{p}\) contributes to wage income and fiscal revenues at the cost of the environment. For the stable, undesirable steady state (i.e., the EPT), an increase in \(\bar{p}\) is to the detriment of both the environment and the economy. Due to the low level of capital, public environmental maintenance does not keep up with the increase in the flow of pollution. The environment worsens and agents’ longevity is reduced. Agents respond by saving less, thus further increasing the stock of pollution because fewer resources are available for environmental maintenance. In contrast, an increase in \(\bar{p}\) improves the conditions of the environment and the economy in the desirable steady state. Because of the high level of capital, the increase in the flow of pollution is counterbalanced by even more environmental maintenance. The environment is ameliorated, and agents’ longevity increases and save more. Our results echo the findings by Ono (2002) that decreases in pollution permits could lead to lower capital and environmental quality in the long run. The rationale of Ono (2002) is that there is a critical value for pollution permits at which the positive income effect is balanced by the negative one, such that both the long-run capital and environmental quality are maximized. The rationale behind our result is different and instead depends on the type of

\(^{10}\) The equilibria must lie in the first quadrant, and must be either stable or saddle such that they can be reached.
equilibrium. When multiple equilibria emerge, both steady states entail competing effects on the environment and thus on longevity through the flow of pollution versus public environmental maintenance; how pollution permits alter the relative magnitudes of the competing effects varies depending on the type of equilibrium in which the economy is operating.

The effects of \( \gamma \), \( \eta \), and \( \theta \) on the undesirable steady state are as expected, but the effects on the desirable steady state seem counter-intuitive. To understand the seemingly counter-intuitive results, consider the transition path to the steady state after an increase in environmental maintenance efficiency, \( \gamma \). The new saddle path requires that individuals must be encouraged to save less, such that the level of capital in the next period is lower. The lower level of capital reduces environmental maintenance expenditure, so the steady-state stock of pollution increases in spite of the more efficient environmental maintenance. As the economy transitions to the new steady state, the level of capital is lower and the stock of pollution is higher, such that longevity and period welfare are reduced. Arguments for the other two parameters, \( \eta \) and \( \theta \), are similar.

Table 3 summarizes how the level of capital, the stock of pollution, longevity, and period welfare in the steady state change with the fundamental parameters under green taxes. A higher value for the green-tax rate \( q \) (i.e., a more stringent environmental regulation) is conducive to both the economy and the environment in the long run. The intuition is that a higher value for \( q \) imposes two opposite effects on the steady-state level of capital. A higher \( q \) imposes a positive effect by enhancing agents’ longevity through the reduced stock of pollution, but imposes a negative effect by reducing income per capita. With the benchmark parameters, the positive effect of a higher \( q \) outweighs the negative effect, and thus the steady-state level of capital increases in \( q \). It follows that both steady-state longevity and period welfare increase. However,
a higher green-tax rate does come with a short-run cost along the transition path. In the period following a rise in the green-tax rate \( q \), the level of capital and period welfare temporarily decrease. Therefore, an increase in the green-tax rate harms the economy in the short run but benefits the economy in the long run.

Although the signs associated with \( \gamma \) and \( \theta \) are as expected, the long-run effects of \( \eta \) seem counter-intuitive. An increase in \( \eta \) decreases both the level of capital and the stock of pollution in the steady state. The reason is that for any given stock of pollution, a higher value for \( \eta \) decreases the steady-state level of capital because agents’ longevity is reduced by more harmful pollution and they save less. A lower level of capital in turn decreases the marginal product of pollution. Firms respond by polluting less until the marginal product of pollution matches the green-tax rate again. The negative effect of a lower level of capital outweighs the positive effect of less pollution, leading to decreases in both longevity and welfare.

The comparison of Table 2 and Table 3 reveal the following two results. First, for a more stringent environmental control (a smaller \( \bar{p} \) and a larger \( q \)), the long-run impacts on the stable, undesirable steady state under pollution permits are the same as those on the unique steady state under green taxes. Second, under pollution permits, a less stringent environmental policy (i.e., a larger \( \bar{p} \)) is welfare improving for the desirable steady state. The intuition for the last result is that developed economies are better off to pollute more, grow the economy, and invest in the necessary infrastructure to clean up the environment.

5.3 Sensitivity Analysis of Multiple Equilibria under Pollution Permits

The second column of Table 2 shows the ranges of each parameter that lead to the emergence of multiple equilibria while holding other parameters constant. This section further investigates how
sensitive the emergence of multiple equilibria is under pollution permits to different combinations of policy and technical parameters.

Table 4 shows the combinations of the dissipation rate of the stock of pollution and the number of pollution permits (\(\theta\) on the horizontal axis and \(\bar{p}\) on the vertical axis of each graph) under different sets of technical parameters (\(\eta\) and \(\gamma\)) that give rise to multiple equilibria under pollution permits. First, all else equal, as the value for \(\theta\) increases, the range of \(\bar{p}\) that gives rise to multiple equilibria first expands and then shrinks. And there exists a positive correlation between \(\theta\) and \(\bar{p}\) that can generate multiple equilibria. Second, a larger value for \(\gamma\) allows for a higher possibility of multiple equilibria. This observation is consistent with Proposition 1. Third, the emergence of multiple equilibria is not sensitive to the change in the value for \(\eta\). Overall, multiple equilibria emerge under a wide range of values for the parameters.

### 6. Conclusions and Discussion

In the basic two-period OLG model with standard assumptions regarding technology, preferences, and dynamics for the stock of pollution, we show that pollution permits may give rise to multiple equilibria whereas green taxes do not. Under pollution permits, one nontrivial equilibrium is an EPT acting as a “sink”. The other nontrivial equilibrium is a desirable one exhibiting saddle stability. A developing economy implementing pollution permits might be pulled into an EPT unless the government encourages agents to increase savings to the level on the saddle path. In contrast, green taxes give rise to a unique stable equilibrium, such that an EPT can be avoided. A developing economy implementing green taxes will always converge to the unique equilibrium and the green-tax rate can be chosen to replicate the desirable equilibrium under pollution permits.
Along the transition path, a well-designed environmental regulation with pollution permits can generate approximately the same discounted sum of welfare as under green taxes. Developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions but proceed with caution given the possibility of being drawn into an EPT, because multiple equilibria under pollution permits may emerge for a wide range of parameters. The main findings are robust to an alternative model where agents can actively engage in private healthcare efforts to prolong longevity.

In practice, both pollution permits and green taxes have been used in developed and developing countries. Both policies have won the favor of economists to deal with environmental deterioration (Stavins, 1998). Pollution permits appeal to economists because the price of pollution permits provides the correct incentive for polluters to efficiently decide pollution levels (Hanley, Shogren, & White, 2007). In 1995, the United States launched the Acid Rain Program to control SO₂ and NOₓ, and in January 2015, 28 U.S. states activated the Cross-State Air Pollution Rule to reduce power plant emissions that contribute to ozone and/or fine particle pollution in other states (EPA, 2015). The U.S. has also launched water quality credit trading programs in several states (Borisova & Roka, 2010). In 2005, the European Union initiated the Emissions Trading System to control CO₂ (European Commission, 2015). Other developed countries, such as Switzerland (2008), New Zealand (2010), Japan (2010), Canada (2013), and South Korea (2015), have established their regional or national carbon emission trading systems (Serre et al., 2015). There is, however, a paucity of cap-and-trade policies in force in the developing world. In 2011, China announced seven experimental markets of regional carbon trading and will implement its national Emissions Trading Scheme in 2017 to control CO₂ emissions (The Guardian, 2015). Kazakhstan in 2013 initiated a carbon emission trading system.
Further, developing countries including Mexico, Brazil, Chile, Ukraine, Russia, Turkey, Vietnam, and Thailand are considering carbon emission trading systems (Serre et al., 2015). These examples indicate that environmental regulations with pollution permits have been well established in developed countries, targeting a wide spectrum of pollutants. Developing countries, however, are just in their initial stages of implementing pollution permits, and the targeted pollutant is confined to carbon dioxide. As the popularity of pollution permits grows, it is likely that more developing countries will implement pollution permits to target a broader range of pollutants other than just carbon dioxide. We note that pollutants differ in their dissipation rates, in their threats posed on longevity, and in their difficulty to be cleaned up. These concepts are captured in our model by the select parameters, which allow us to relate our findings to the real world. Our findings are thus likely to have increasing future relevance and can serve as a guide for developing countries to avoid the pitfalls of being drawn into EPTs.

Green taxes also appeal to economists because green taxes create incentive for polluters to reduce emissions (Hanley et al., 2007). The U.S. use a few green taxes, whereas many European countries impose green taxes on a variety of air pollutants such as SO₂ and NOₓ (Levinson, 2007). Developing countries such as Chile and Mexico have chosen to implement a carbon tax (Serre et al., 2015). Although no EPT emerges under green taxes within our model, developing countries using green taxes still need to carefully choose the tax rate, such that the long-run outcome is comparable to the desirable equilibrium under pollution permits.

We close by noting that another potential way to avoid the EPT under pollution permits is to endogenously set up the quantity restriction for pollution. In our framework, the number of pollution permits is exogenously determined. Although we have shown that an exogenous gradual change in pollution permits across generations may not eliminate the EPT, future
research should explore more fully how robust the EPT is to a pollution quota that is allowed to endogenously move within and across generations.
Figure 1. Phase Diagram and Multiple Equilibria under Pollution Permits
Figure 2. Phase Diagram under Green Taxes
Figure 3. Comparison of Transition Paths with Government Intervention under Pollution Permits
Figure 4. Comparison of Transition Paths without Government Intervention
Period 0: $\bar{p} = 5$

Period 1: $\bar{p} = 4.9$

Period 2: $\bar{p} = 4.802$

Period 3: $\bar{p} = 4.70596$

Figure 5. The Effects of Decreases in Pollution Permits (Decreases by 2% Each Period)
Table 1. Descriptions and Values for Benchmark Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Capital’s share in production</td>
<td>$\alpha$</td>
<td>0.35*</td>
</tr>
<tr>
<td>Labor’s share in production</td>
<td>$\beta$</td>
<td>0.63*</td>
</tr>
<tr>
<td>Production function scalar</td>
<td>$B$</td>
<td>10</td>
</tr>
<tr>
<td>Speed of pollution dissipation</td>
<td>$\theta$</td>
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</tr>
<tr>
<td>Environmental maintenance efficiency</td>
<td>$\gamma$</td>
<td>15**</td>
</tr>
<tr>
<td>Lower bound of longevity</td>
<td>$\lambda$</td>
<td>0</td>
</tr>
<tr>
<td>Upper bound of longevity</td>
<td>$\overline{\lambda}$</td>
<td>1***</td>
</tr>
<tr>
<td>Detrimental effect of pollution on health</td>
<td>$\eta$</td>
<td>25</td>
</tr>
<tr>
<td>Number of pollution permits</td>
<td>$\overline{p}$</td>
<td>5****</td>
</tr>
<tr>
<td>Green-tax rate</td>
<td>$q$</td>
<td>0.0572****</td>
</tr>
<tr>
<td>Objective discount factor</td>
<td>$\delta$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Notes.** * Brock and Taylor (2005) argue that pollution’s share in the production function should be between 0.01 and 0.02. We follow the illustrative example in Brock and Taylor (2005), and set capital’s share $\alpha = 0.35$ and pollution’s share $1 - \alpha - \beta = 0.02$, implying labor’s share $\beta = 0.63$.

** The environmental maintenance efficiency also serves as a conversion parameter that transforms the resources of environmental maintenance in the unit of final output into pollution reduction in the unit of pollution.

*** The life expectancy at birth in Japan is 83.3 years in 2013, which is the longest in the world (World Bank Group, 2015b). If each period is 50 years, $\overline{\lambda} = 1$ implies that each generation’s potential lifetime longevity is 100 years.

**** The value for $\overline{p}$ is chosen such that in the desirable steady state under pollution permits, longevity is approximately 0.39, implying a lifetime longevity of $50 \times (1 + 0.39) = 69.5$ years. This value for the lifetime longevity roughly matches the average life expectancy at birth in countries whose data are available. In 2013, the average life expectancy at birth is 70.8 years across the countries (World Bank Group, 2015b).

***** The value for green-tax rate corresponds to $\overline{p}$. The green-tax rate is set such that the unique steady state under green taxes overlaps the desirable steady state under pollution permits. The digits after the decimal point are rounded. Also note that the parameter restriction $1 - \gamma q > 0$ is satisfied such that a nontrivial steady state exists under green taxes.
Table 2. Steady-State Effects of Parameter Increases under Pollution Permits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range#</th>
<th>Undesirable Steady State</th>
<th>Desirable Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$k_{ss}$</td>
<td>$z_{ss}$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>(-9%, 17%)</td>
<td>-*</td>
<td>+</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(-14%, 18%)</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(-21%, 96%)</td>
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<td>+</td>
</tr>
<tr>
<td>$\theta$</td>
<td>(-100%, 25%)</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes. *The ranges of each parameter allow for multiple equilibria while holding other parameters constant. The ranges are expressed as percentage deviations from the benchmark values listed in Table 1.

* The signs are ambiguous in comparative statics performed in Appendix D, and are obtained using numerical simulations based on benchmark values in Table 1. The unambiguous signs derived in Appendix D can also be verified by the numerical results.
Table 3. Steady-State Effects of Parameter Increases under Green Taxes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k_{ss}$</th>
<th>$z_{ss}$</th>
<th>$\phi_{ss}$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>+*</td>
<td>-</td>
<td>+*</td>
<td>+*</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>+</td>
<td>-</td>
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<td>+</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes. * The signs are ambiguous in comparative statics performed in Appendix D, and are obtained using numerical simulations based on benchmark values in Table 1. The unambiguous signs derived in Appendix D can also be verified by the numerical results.
Table 4. Sensitivity Analysis of Multiple Equilibria under Pollution Permits

<table>
<thead>
<tr>
<th>Detrimental Effect of Pollution on health, $\eta$</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
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<tr>
<td>20</td>
<td>![Graph 1]</td>
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<td>![Graph 3]</td>
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<td>25</td>
<td>![Graph 4]</td>
<td>![Graph 5]</td>
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<td>30</td>
<td>![Graph 7]</td>
<td>![Graph 8]</td>
<td>![Graph 9]</td>
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</table>

Environmental Maintenance Efficiency, $\gamma$
References

http://edis.ifas.ufl.edu/pdfiles/FE/FE82400.pdf


*Environmental and Resource Economics, 21*(1), 75-87.


