Home work # 1
Math 5290-Fall 2008
Due date 09/22/08 in class

1. **(ex2.6 pp12).** Let $\mathcal{A}$ be a σ-algebra of subsets of Ω and let $B \in \mathcal{A}$. Show that $\mathcal{F} = \{A \bigcap B : \ A \in \mathcal{A}\}$ is a σ-algebra of subsets of B. Is it still true when B is a subset of Ω that does not belong to $\mathcal{A}$?

2. **(ex3.9 pp19).** Suppose that A, B, C are independent events and that $P(A \bigcap B) \neq 0$. Show that

$$P(C|A \bigcap B) = P(C).$$

3. **(ex5.20 pp34).** Suppose that a r.v. X takes its values in $\mathbb{N}$. Show that

$$E[X] = \sum_{n=0}^{\infty} P(X > n).$$

4. **(ex7.18 pp46).** Suppose that a function F is given by

$$F(x) = \sum_{i=1}^{\infty} \frac{1}{2^i} 1_{[\frac{1}{i},\infty)}(x).$$

   Show that it is a distribution function.

   Let us define P a probability distribution by

$$P((-\infty, x]) := F(x).$$

   Find the probabilities of the following events

   (a) $A = [1, \infty)$.
   (b) $B = [1/10, \infty)$.
   (c) $C = 0$.
   (d) $A = (-\infty, 0)$.

5. **(Bonus question)**

(ex3.17 pp20). Let $A_1, \ldots, A_n$ be independent events. Show that the probability that none of the $A_1, \ldots, A_n$ occur is less or equal to

$$\exp\left(-\sum_{i=1}^{n} P(A_i)\right).$$