Home work # 3
Math 5290-Fall 2008
Due date 10/08/08 in class

1. **(ex14.9 pp115).** Let $X_1, \ldots, X_n$ be independent, each with mean 0, and each with finite third moments. Show that

$$E\left\{\left(\sum_{i=1}^{n} X_i\right)^3\right\} = \sum_{i=1}^{n} E\left\{X_i^3\right\}.$$

(Hint: Use characteristic functions.)

2. **(ex14.13 pp115).** Let X be a positive random variable. The Mellin transform of X is defined to be

$$T_X(\theta) = E\left\{X^\theta\right\}$$

for all values of θ for which the expected value of X^θ exists.

   (a) Show that
   $$T_X(\theta) = \varphi_{\ln(X)}\left(\frac{\theta}{i}\right)$$
   when all terms are well defined.

   (b) Show that if X and Y are independent and positive, then
   $$T_{XY}(\theta) = T_X(\theta)T_Y(\theta).$$

   (c) Show that
   $$T_{bX^a}(\theta) = b^\theta T_X(a\theta)$$
   for $b > 0$ and $a\theta$ in the domain of definition of T_X.

3. **(ex14.15 pp116).**

   Let X be $\mathcal{N}(0,1)$. Show that

   (a) $E\left\{X^{2n+1}\right\} = 0.$

   (b) $E\left\{X^{2n}\right\} = \dfrac{(2n)!}{2^n n!} = (2n-1)(2n-3)\ldots 3 \cdot 1.$

4. **(ex17.6 pp148).**

   Let $X_n \longrightarrow^P X$. Show that the characteristic functions φ_{X_n} converge pointwise to φ_X.

   Hint: Use Theorem 17.4.

5. **(ex17.10 pp148).** Let X_j be i.i.d. with finite variances and zero means. Let

$$S_n = \sum_{i=1}^{n} X_j.$$

   Show that $\frac{1}{n}S_n$ tends to 0 in both L^2 and in probability.