Home work # 5
Math 5290-Fall 2008
Due date 11/10/08 in class

1. **(ex1.1.2 pp8).** Suppose that $(X_n)_{n\geq 0}$ is Markov (λ, P). If $Y_n = X_{nk}$, show that $(Y_n)_{n\geq 0}$ is Markov (λ, P^k).

2. **(ex1.1.3 pp8).** Let X_0 be a random variable with values in a countable set I. Let $Y_1, Y_2, \ldots$ be a sequence of independent random variables, uniformly distributed on [0,1]. Suppose we are given a function

$$G : I \times [0,1] \longrightarrow I$$

and define inductively

$$X_{n+1} = G(X_n, Y_{n+1}).$$

Show that $(X_n)_{n\geq 0}$ is a Markov chain and express its transition matrix P in terms of G. Can all Markov chains be realized in this way? How would you simulate a Markov chain using a computer?

Suppose now that $Z_0, Z_1, \ldots$ are independent, identically distributed random variables such that $Z_i = 1$ with probability p and $Z_i = 0$ with probability $1-p$. Set

$$S_0 = Z_1 + Z_2 + \cdots + Z_n.$$

In each of the following cases determine whether $(X_n)_{n\geq 0}$ is a Markov chain:

(a) $X_n = Z_n$

(b) $X_n = S_n$

(c) $X_n = S_0 + \cdots + S_n$

(d) $X_n = (S_n, S_0 + \cdots + S_n)$

In the cases where $(X_n)_{n\geq 0}$ is a Markov chain find its state-space and transition matrix, and in the cases where it is not a Markov chain give and example where $P(X_{n+1} = i | X_n = j, X_{n-1} = k)$ is not independent of k.

3. **(ex1.2.1 pp11).** Identify the communicating classes of the following transition matrix

$$P = \begin{pmatrix} 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 0 & 0 & 1/2 \end{pmatrix}$$

Which classes are closed?