Homework # 7
Math 5200-Spring 2009
Due date April 24th 2009.

1. Let
$$f(x) = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$$
where $f(x)$ has period 2π. Obtain the Fourier series for $f(x)$ and draw the graph of f.

2. Prove that if $f(x)$ is an odd function in $(-l, l)$ $\big(f(-x) = -f(x)\big)$, then the Fourier series corresponding to $f(x)$ is
$$\sum_{n=1}^{\infty} b_n \frac{\sin n\pi x}{l} \text{ where } b_n = \frac{2}{l}\int_0^l f(x)\frac{\sin n\pi x}{l}dx.$$
The series is often called a Fourier sine series.

3. Prove that
$$\lim_{n\to\infty}\int_0^\pi \frac{\sin nx}{x^2+1}dx = 0$$
in two different ways.

4. Let $f(x) \in L^2[-\pi, \pi]$ have the Fourier series
$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\big(a_n \cos nx + b_n \sin nx\big).$$
Prove that Parseval's identity is
$$\frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) = \int_{-\pi}^{\pi} |f(x)|^2 dx$$
and thus prove that the series on the left converges.