

Mathematics 4255

Midterm 1

February 19, 2009

Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!

1	
2	
3	
4	
5	
6	
Total	

Name: _____

I. **10points** Two dices are thrown. What is the probability that the total number of dots is

1. equal 7

2. an even number

II. **10points** Let us denote by S a sample set and suppose that \mathcal{A} is the family of all subsets of S . We fix $x \in S$ and define a set-function

$$\delta_x(A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

1. Show that δ_x is a probability measure.
2. What is the largest set of measure 0? What is the largest set of measure 1.

III. **10points** Groping about in the dark, we open one of the two drawers in a chest and pick up an item of clothing at random.

1. What is the probability that it is a sock if one drawer contains 6 socks and 6 underpants and the other contains 2 socks and 4 handkerchiefs.
2. What is the probability that the item comes from the first drawer if it turns out to be a sock.

VI. **10 points** Prove the following statements:

1. If $P(A) = 0$, then what is $P(A \cap B)$?
2. If $P(A) = 1$, then what is $P(A \cap B)$?

V. **10 points** Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are

1. H, H, H, H.
2. T, H, H, H.

VI. (**Bonus 10 points**) Let $S = \{1, 2, \dots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including the null set and S itself) of S .

1. Show that

$$P(A \subset B) = \left(\frac{3}{4}\right)^n.$$

Hint: Let $N(B)$ denote the number of elements in B . Use

$$P(A \subset B) = \sum_{i=0}^n P(A \subset B | N(B) = i) P(N(B) = i).$$

2. Show that

$$P(A \cap B = \emptyset) = \left(\frac{3}{4}\right)^n.$$