

# Mathematics 4255

## Midterm 2

April 2nd, 2009

Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!

<b>1</b>	
<b>2</b>	
<b>3</b>	
<b>4</b>	
<b>5</b>	
<b>Total</b>	

Name: \_\_\_\_\_

I. (10points) The joint density function of  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \textit{otherwise} \end{cases}$$

1. Are  $X$  and  $Y$  independent?
2. Find  $P(X + Y < 1)$ .

II. (**10points**) Assume  $X$  and  $Y$  are independent exponential random variables with common parameter  $\lambda$ .

1. Find the probability density function of  $Z = X + Y$ .
2. Find  $P(Y < t | X + Y > s)$ , for some  $s > 0$  and  $t > 0$ .

**Hint:** Recall that  $X$  is an exponential random variable with parameter  $\lambda$  if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

III. **(10points)** Let  $X$  be a normal random variable with mean 12 and variance 4. Find the value of  $c$  such that  $P(X > c) = 0.10$ .

VI. (10 points) The joint probability mass function of the random variable  $X, Y, Z$  is given by

$$p(1, 2, 3) = p(2, 2, 1) = p(2, 3, 2) = \frac{1}{3}.$$

Find  $E[XYZ]$ .

V. (10 points)

1. Let  $X$  be uniformly distributed over  $(0, 1)$ . Let  $Y := X^n$ ,  $n \geq 1$  is an integer. Find the Probability density function (pdf) of the r.v  $Y$ .
2. Let  $X$  be a continuous random variable with probability density function  $f_X$ . Let  $Y := X^2$ . Find the probability density function of the r.v.  $Y$ .