1. Let $f$ be a real-valued function defined as

$$f(x) = \begin{cases} 
  x^2 \sin \frac{\pi}{x} & x \neq 0 \\
  0 & x = 0 
\end{cases}$$

Prove that $f$ is differentiable but does not belong to $BV[0,1]$.

2. Let $F$ be monotone increasing on $[a, b]$. Find $T_F[a, b]$, the total variation of $F$ on $[a, b]$.

3. Show that if there is a Lipschitz constant $M > 0$ such that

$$|F(x) - F(y)| \leq M|x - y| \quad \forall x, y \in [a, b]$$

then $F \in BV[a, b]$. 