1. Let $m^*$ be an outer measure on a set $X$. Let $A$ be a set of measure zero. Show that 

$$m^*(B \cup A) = m^*(B \setminus A) = m^*(B)$$

holds for every subset $B$ of $X$.

2. (a) What does it mean to say that $f$ is Lebesgue measurable on $\mathbb{R}$?
   
   (b) Prove that if a function is continuous a.e. (almost everywhere) on $\mathbb{R}$ then it is Lebesgue measurable on $\mathbb{R}$?

3. Compute the following limit if it exists

$$\lim_{n \to \infty} \int_0^\infty \left(1 + \frac{x}{n}\right)^{-n} \sin \left(\frac{x}{n}\right) dx$$

4. Let $I = [0, 1]^2$ and let us define a real-valued function $f$ on $I$ by

$$f(x, y) = \frac{x^2 - y^2}{(x^2 + y^2)^2}, \quad x, y \in I.$$ 

Prove that $f \notin L^1(I)$. 