Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!
I. (10 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function given by $f(x) = x^2 + 1$.

1. Find the range of $f$.

2. Is $f$ a one to one (injective) function? Justify your answer.

3. Is $f$ an onto (surjective) function? Justify your answer.
II. (10 points) For each subset of $\mathbb{R}$, give its supremum, maximum, infimum and minimum, if they exist

1. $\{r \in \mathbb{Q}, r < 5\}$,

2. $\left\{ (-1)^n \left( 1 + \frac{1}{n} \right), \ n \in \mathbb{N} \right\}$. 

III. **(10 points)** Let \((s_n)_n\) be a sequence defined by induction as follows:

\[
\begin{align*}
s_1 &= 1 \\
 s_{n+1} &= \frac{1}{4}(s_n + 5) \quad n \in \mathbb{N}
\end{align*}
\]

1. Prove that \((s_n)_n\) is a monotone and bounded sequence.

2. Find its limit.
VI. **10 points** Let \((s_n)_n\) a sequence given by \(s_n = n \left(\sin \left(\frac{n\pi}{2}\right)\right)^2\).

1. Write all the possible subsequences.
2. Find the limit superior.
3. Find the limit inferior.
V. (Bonus 10 points) A sequence \((s_n)_n\) is said to be contractive if there exists a constant \(k\) with \(0 < k < 1\) such that

\[
|s_{n+2} - s_{n+1}| \leq k|s_{n+1} - s_n| \quad \forall n \in \mathbb{N}.
\]

Prove that every contractive sequence is a Cauchy sequence and hence is Convergent.