1. (i) Show that a countable union of sets of measure zero is again a set of measure zero.
(ii) A set $U$ is said to be a $G$-set if $U$ is the intersection of a countable collection of open sets. Prove that if $A$ has measure zero then there exists a $G$-set $U$ of measure zero such that $A \subseteq U$.

2. Let $f$ be a function with measurable domain $D$. Show that $f$ is measurable if and only if the function $g$ defined by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.

3. Compute the following limit if it exists

$$\lim_{n \to \infty} \int_0^\infty n \sin \frac{x}{n} (x(1 + x^2))^{-1} \, dx$$

4. Show the following

(a) Let $f$ be integrable over $(-\infty, \infty)$, then

$$\int f(x) \, dx = \int f(x + t) \, dx, \quad t \in (-\infty, \infty)$$

(b) Let $g$ be a bounded measurable function, then

$$\lim_{t \to 0} \int |g(x)| |f(x) - f(x + t)| \, dx = 0.$$ 

5. Prove the following variant of Fatou’s Lemma: If $\{f_n\}$ is a sequence of non-negative measurable functions which converges to $f$ a.e. and $\int_X f_n(x) \, dx \leq M < \infty$ for all $n$, then $f$ is integrable and $\int_X f(x) \, dx \leq M$. 