Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!
I. (10pts) In a certain college, 25% of the students failed mathematics, 15% of the students failed chemistry and 10% of the students failed both mathematics and chemistry. A student is selected at random.

1. If he failed chemistry, what is the probability that he failed mathematics?
2. If he failed mathematics, what is the probability that he failed chemistry?
3. What is the probability that he failed mathematics or chemistry?
II. (10pts) Let $X$ and $Y$ be independent random variables taking values in the positive integers and having the same mass function $f(k) = \frac{1}{2^k}$ for $k = 1, 2, \ldots$. Find

1. $P(\min \{X, Y\} \leq k)$.
2. $P(Y > X)$.
3. $P(X > kY)$ for a given positive integer $k$. 
III. (10pts)
Let $X$ be a discrete random variable such that

$$X(\omega) = \begin{cases} 1 & \omega \in A \\ 2 & otherwise \end{cases}$$

1. Find the distribution function of $X$ if $P(A) = \frac{1}{3}$.
2. Find the expected and the variance of $X$. 
VI. (10pts) Suppose that $X$ and $Y$ are jointly continuous random variables with joint density

$$f_{X,Y}(x, y) = \begin{cases} 
  ce^{x+y} & (x, y) \in (-\infty, 0] \times (-\infty, 0] \\
  0 & \text{otherwise}
\end{cases}$$

1. what is the value of $c$?

2. What is the probability that $X < Y$?

3. What are the marginal densities $f_X$ and $f_Y$?
V. (10pts) If $X$ is a Poisson random variable with parameter $\lambda$, show that

1. 

$$E[X^n] = \lambda E[(X + 1)^{n-1}] .$$

2. Now use this result to compute $E[X^3]$. 
VI. (5pts (Bonus)) \( X \) is a Poisson random variable with parameter \( \lambda \),
Show for \( r = 2, 3, \ldots \),
\[
E [X(X - 1) \ldots (X - r + 1)] = \lambda^r.
\]