Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!
I. (10 pts) Find the solution of the given initial value problem below

\[ y'' + y' - 2y = 0 \]
\[ y(0) = 1, \quad y'(0) = 1. \]
II. (10 pts)

1. Determine $\omega_0$, $R$ and $\delta$ such that

$$u(t) = e^{-4t}(-2\cos \pi t - 3\sin \pi t)$$

can be written in the following form

$$u(t) = Re^{-4t}\cos(\omega_0 t - \delta).$$

2. Find the limit of $u(t)$ when $t \to \infty$. 
III. (10 pts)
Find the general solution of the following problem

\[ y'' + 2y' + y = 2e^{-t} \]
\[ y(0) = 1, \ y'(0) = 1. \]
IV. (10 pts)
For $x > 0$, let the following differential equation be given

$$x^2 y'' - 3xy' + 4y = x^2 \ln x$$

1. Prove that $\{x^2, x^2 \ln x\}$ is a set of fundamental solutions.

2. Using the variation of parameters, find the general solution of the above differential equation.