Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!
I. (10 pts)
Find the solution of the given initial value problem below

\[ y'' + 2y' + y = 2e^t \]
\[ y(0) = 1, \quad y'(0) = 1. \]
II. (10 pts)
Consider the forced but undamped system described by the initial value problem

\[ u'' + 2u = \cos t \]
\[ u(0) = 0, \ u'(0) = 0. \]

1. Find the solution \( u(t) \).
2. Rewrite the solution \( u(t) \) in the following form of a product of two sin functions.
3. Find the period of the solution \( u(t) \).
III. (10 pts)

1. Determine $\omega_0$, $R$ and $\delta$ such that

$$u(t) = e^{-4t}(-2\cos\pi t - 3\sin\pi t)$$

can be written in the following form

$$u(t) = Re^{-4t}\cos(\omega_0 t - \delta).$$

2. Find the limit of $u(t)$ when $t \to \infty$. 

IV. Reminder about Trigonometry

1. $\sin(x + y) = \sin x \cos y + \cos x \sin y$,
2. $\sin(x - y) = \sin x \cos y - \cos x \sin y$,
3. $\cos(x + y) = \cos x \cos y - \sin x \sin y$,
4. $\cos(x - y) = \cos x \cos y + \sin x \sin y$. 