Partial credit will be awarded for your answers, so it is to your advantage to explain your reasoning and what theorems you are using when you write your solutions. Please answer the questions in the space provided and show your computations.

Good luck!
1. (10 points) Classify each of the following sets as open, closed, neither or both (justify your answer)

   1. \( \left\{ \frac{1}{n}, \ n \in \mathbb{N} \right\} \)

   2. \( \left\{ x, \ |x - 5| \leq 2 \right\} \)

   3. Find the closure of each of the previous sets
II. (10 points) Using the $\epsilon$-$\delta$ definition, prove that the function $f(x) = x^2 + 2x + 7$ is uniformly continuous on $[0,1]$.
III. (10 points) Determine whether each of the following series converges or diverges. Justify your answer.

1. \[ \sum \frac{1}{n\sqrt{n + 1}} \]

2. \[ \sum \frac{\sin^2 n}{n^2} \]
VI. (10 points) Let

\[ f(x) = \frac{(x^2 - 4x - 5)}{(x - 5)}, \quad \text{for} \quad x \neq 5. \]

How should \( f(5) \) be chosen so that \( f \) will be continuous at 5?
V. (10 points) Suppose that \( f : [a, b] \rightarrow \mathbb{R} \) and \( g : [a, b] \rightarrow \mathbb{R} \) are continuous functions such that
\[
f(a) \leq g(a), \quad \text{and} \quad f(b) \geq g(b).
\]
Prove that \( f(c) = g(c) \) for some \( c \in (a, b) \).
VI. (Bonus 10 points) Suppose that \((a_n)\) is a sequence of numbers such that for all \(n\),

\[|a_{n+1} - a_n| \leq b_n\]

where \(\sum b_n\) is convergent. Show that \((a_n)\) converges.