1. **(5 pts)** Let \( f \) be a function defined on \( \mathbb{R} \). We define the translation of \( f \) by an \( h \in \mathbb{R} \) as a function \( f_h \) as follows:

\[
f_h(x) := f(x - h), \quad x \in \mathbb{R}.
\]

Using the invariance of the Lebesgue measure, if \( f \in L^1(\mathbb{R}) \) then \( f_h \in L^1(\mathbb{R}) \) and

\[
\int_{\mathbb{R}} f(x - h) \, dx = \int_{\mathbb{R}} f(x) \, dx.
\]

Prove the following result: Assume \( f \in L^1(\mathbb{R}) \). Then

\[
\lim_{h \to 0} \| f_h - f \|_{L^1} = 0.
\]

2. **(5 pts)** Prove the following

(a) If \( f \) is of bounded variation on \([a, b]\) then it is bounded on \([a, b]\).

(b) If \( f \) and \( g \) are two functions of bounded variations on \([a, b]\) then their product \( fg \) is also of bounded variation on \([a, b]\).

3. **(5 pts)** Compute the following limit if it exists

\[
\lim_{n \to \infty} \int_0^{\infty} n \sin \frac{1}{n} (x(1 + x^2))^{-1} \, dx
\]

4. **(5 pts)** Suppose that \( f \in L^1(\mathbb{R}) \). Prove that the sequence \( s_n = \int_{\mathbb{R}} f(x) \cos nx \) converges to 0 as \( n \to \infty \).