Abstract

Turbulence is a property of many fluid flows found in nature (the atmosphere and the ocean, for instance) or in the industry, characterized, among other things, by the coupling of a wide range of length scales and by strong time fluctuations. It affects for instance the ability of the flow to transport energy or tracers.

Although turbulent flows are in principle deterministic, the large number of degrees of freedom and the chaotic nature of the system lead to an unpredictable behavior in practice. Therefore, most theories consider turbulent flows in a probabilistic way, and try to provide statistical predictions for various quantities. This amounts to building a probability measure on phase space. One way to achieve this is to discard completely the details of the microscopic dynamics and build invariant measures based only on a few macroscopic quantities: the dynamical invariants of the inviscid system. This is the purpose of equilibrium statistical mechanics. This program has been applied to a variety of systems, and in particular to Hamiltonian systems, such as the well-known ideal gas, or magnetic systems such as the Ising model, and provides accurate predictions about the macroscopic behavior of the system.

Most of these lectures will be devoted to adapting these methods to turbulent flows. A major difficulty in doing so is the infinite dimensional nature of the phase space of turbulent flows. In a first lecture, I will review the approaches which study the equilibrium properties of finite-dimensional approximations of turbulent flows, and in a second lecture, I will describe the construction of the equilibrium measures for the infinite dimensional system.

We shall see that there are various kinds of turbulent flows. For instance, in physical applications, the dimension of the domain can be 2 or 3. The behavior of these two cases differ dramatically. We shall see that the program sketched above is well-suited for two-dimensional flows, on which these lectures focus, while three-dimensional flows exhibit critical pathologies. We will also discuss some intermediate models, relevant to the study of geophysical flows, such as geostrophic turbulence and rotating-stratified turbulence.

Tentative Outline

1 Introduction

2 Finite-dimensional approximations for 2D turbulence
   2.1 Onsager and the point vortices
   2.2 Kraichnan and the Galerkin-truncated flows
   2.3 Beyond 2D flows: rotating-stratified turbulence and the restricted partition function

3 The Robert-Miller-Sommeria mean-field theory
   3.1 The microcanonical measure
   3.2 The large deviation property and the mean-field approach
   3.3 Solutions of the mean-field equation

4 Summary
   And, if time permits, a word on non-equilibrium statistical mechanics

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