1. Introduction to semimartingale stochastic integration

Stochastic integrals for cadlag integrands and integrators will be introduced as limits of Riemann-like sums. The proof of existence will be sketched for martingale and finite variation integrators (and hence for semimartingales), and Itô’s formula will be introduced.

2. Existence and uniqueness for stochastic differential equations

Stochastic differential equations will be introduced along with a few standard examples. The proof of uniqueness under the standard Lipschitz conditions will be given.

3. Change of measure

Complicated stochastic models can frequently be constructed by changing the measure on a probability space that supports a similar but much simpler model. For stochastic processes, these changes of measure are associated with nonnegative, mean-one martingales. For models determined by martingales, the effects of a change of measure on the models can be understood by first understanding the effects on martingales defined on the probability space.

4. Weak convergence for stochastic integrals and differential equations

Given a sequence of semimartingale integrators and integrands that converges in distribution, when does the sequence of stochastic integrals converge? When and to what do the solutions of a sequence of stochastic differential equations driven by the semimartingales converge?

5. Infinite dimensional semimartingales

Space-time Gaussian white noise and space-time Poisson random measures are two examples of models that can be interpreted as infinite dimensional semimartingales. There are many ways of defining stochastic integrals against these objects. The definition that will be given here makes explicit use of the stochastic integral for finite dimensional semimartingales. This approach allows a complete generalization of the weak convergence results discussed in Lecture 4.

6. Martingale problems and stochastic differential equations for Markov processes

Stochastic differential equations for Markov processes driven by Brownian motion or Gaussian white noise and Poisson random measures will be introduced. The notion of a martingale problem will be described, some of the basic properties of martingale problems discussed, and the proof of the equivalence of martingale problems and stochastic differential equations for Markov processes in \( \mathbb{R}^d \) will be sketched.