$$f = \underbrace{(1,2,4,5,6)}_{\text{even}} \underbrace{(7,8,9)}_{\text{even}}$$

$$(1,6)(1,5)(1,4)(1,2)\ (7,9)(7,8)(1,2)(1,2)(1,2)(1,2)$$

f is even.

$f \circ f$ is always even

$$\underset{\text{odd} \,\circ\, \text{even} \,\circ\, \text{even}}{(1,2)\, f\, (1,2,3)} = \text{odd perm.}$$

Solve

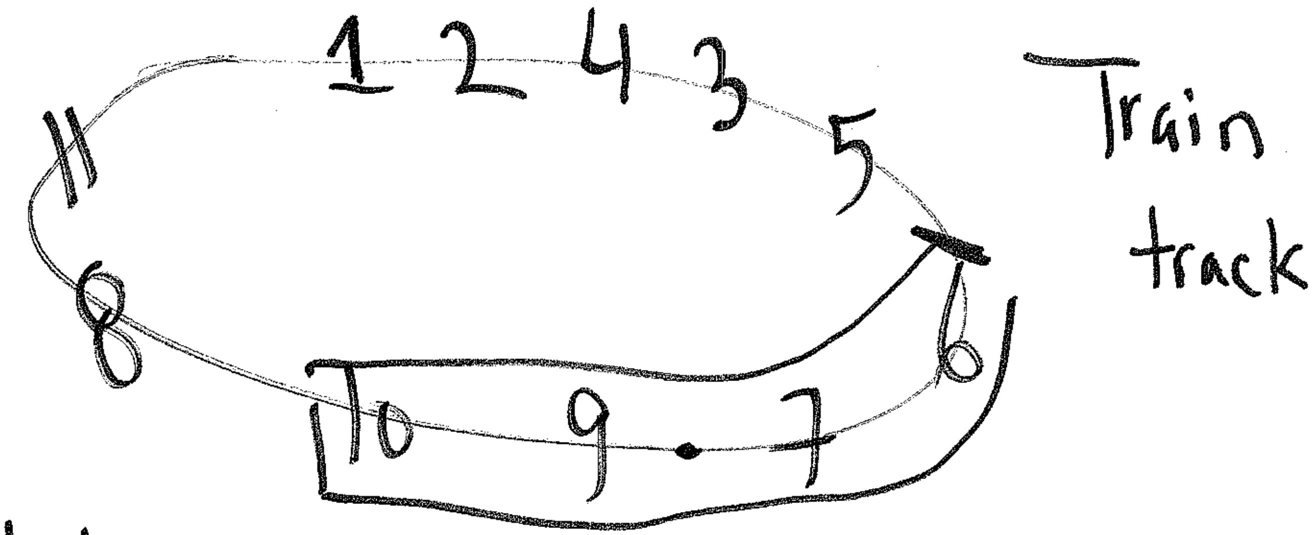
$$\underbrace{g}_{\text{odd}}\ \underbrace{(1,2,3)}_{\text{even}} = \underbrace{(1,2)}_{\text{odd}}$$

$$g\,(1,2,3)(1,3,2) = (1,2)(1,3,2)$$

$$g\,(1,2,3)(1,3,2) = g \circ id = g$$

$$(1,2)(1,3,2) = (1)\,(2\,3)$$

HW like problem:



Rotate train around cw or ccw.
Roundabout allows us to swap 4 cars
Can we get the cars in order?

Problem:

Can we find product P of f's & g's

s.t.

$$P \circ h = \text{identity}$$

So P is even.

even

h must be even.

Basic moves:

Rotate all cars:

$$f = (\underbrace{1, 2, 3, \ldots, 10, 11}_{\text{positions}})$$

f is an even perm

$$g = (14)\ (23)$$

g is even

h = original permutation

$$h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 4 & 3 & 5 & 6 & 7 & 9 & 10 & 8 & 11 \end{pmatrix}$$

$$h = \underbrace{(3\ 4)}_{\text{odd}}\underbrace{(8,9,10)}_{\text{even}} \qquad h \text{ is odd.}$$

So there is no sol'n.

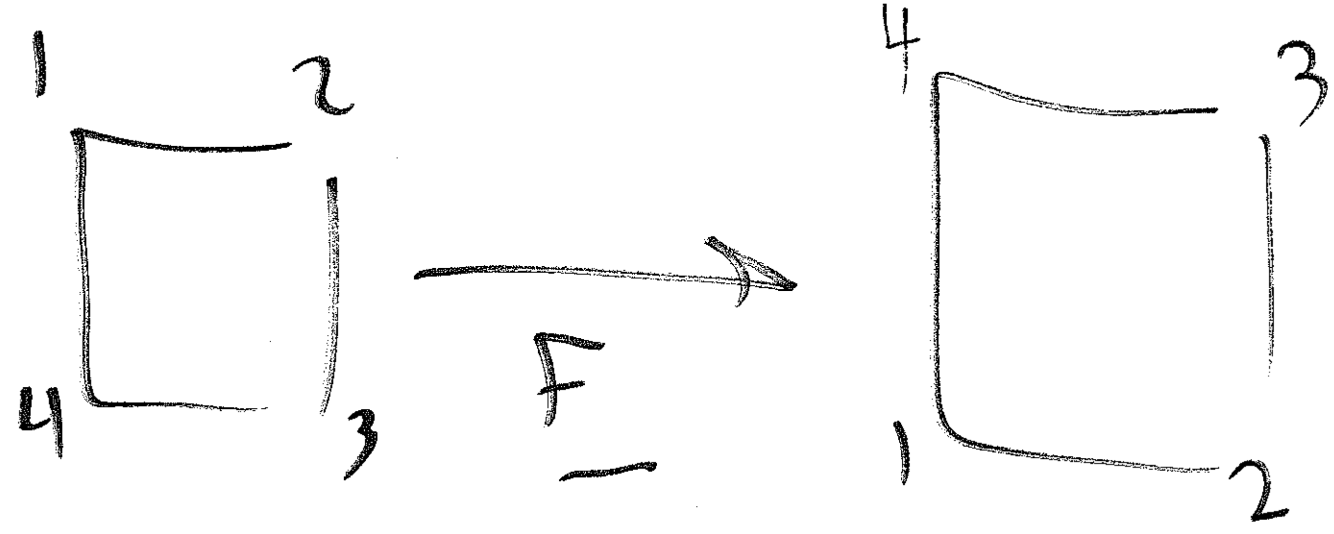
Rigid Motions of Square

1 2

8 rigid motions of Square

4 3

4 choices for bottom side
2 orientations for bottom side.

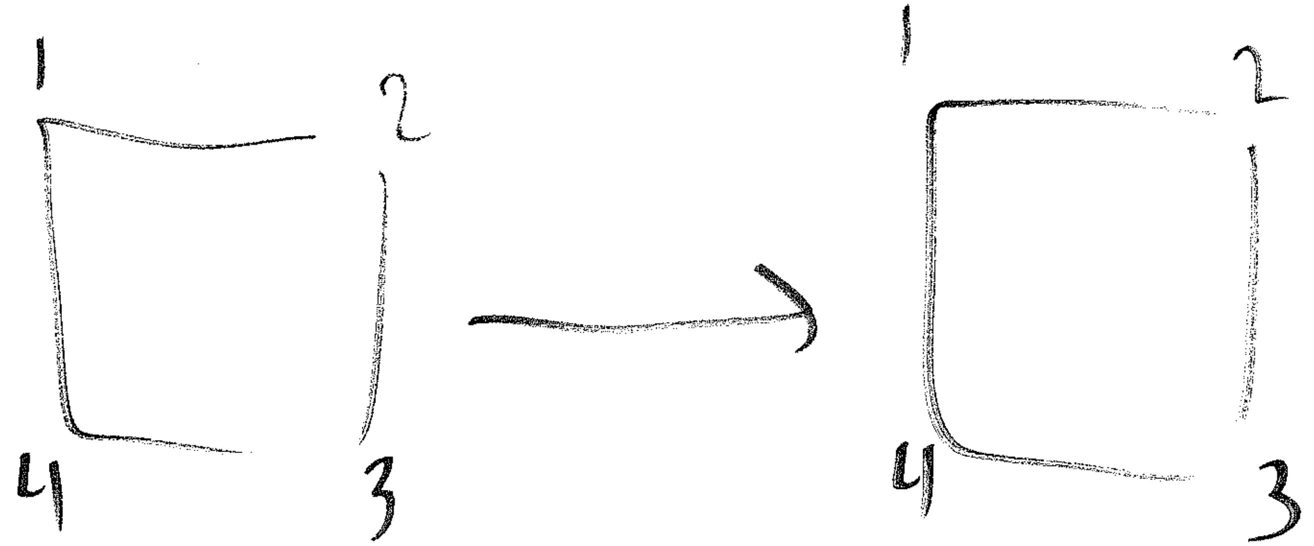


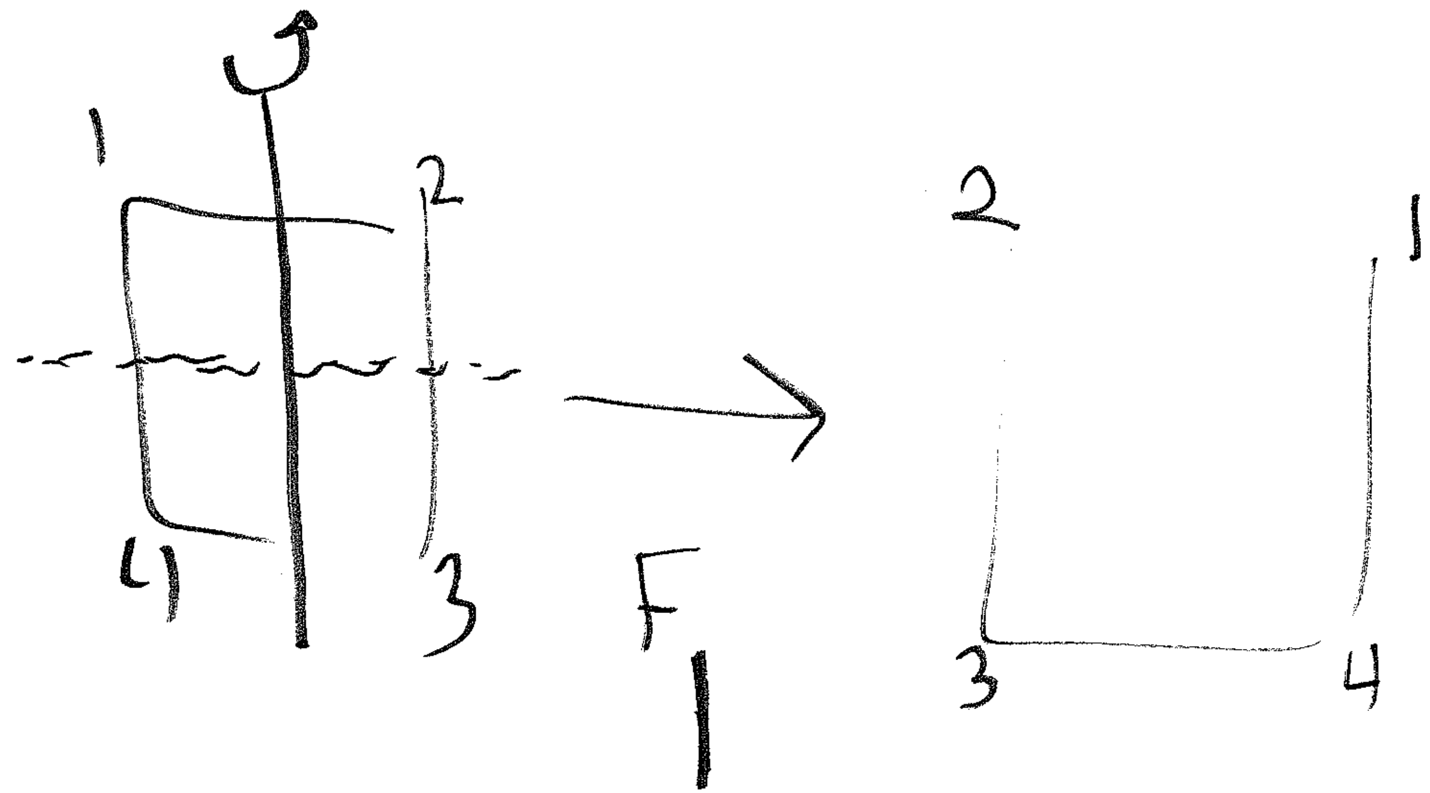
$F_- = (1\,4)(2\,3)$

Even perm

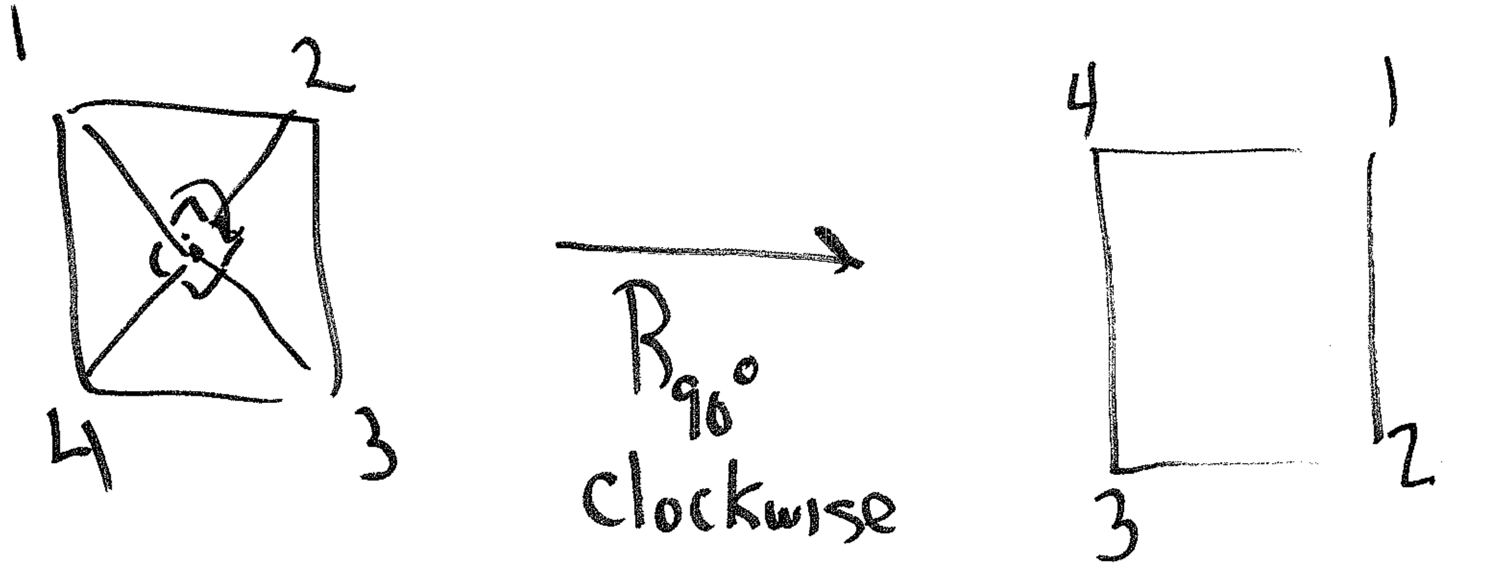
$(F_-)^{-1} = F_-$

identity





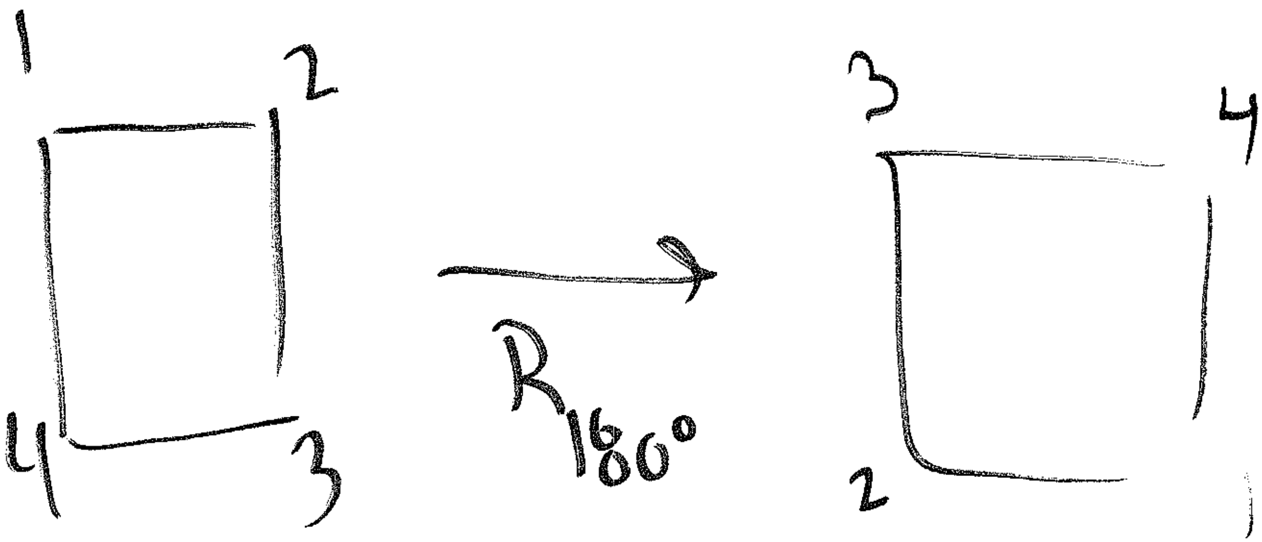
$$F_1 = (1\ 2)(3\ 4)$$



$$R_{90^\circ} = (1,2,3,4) \qquad \text{odd perm}$$

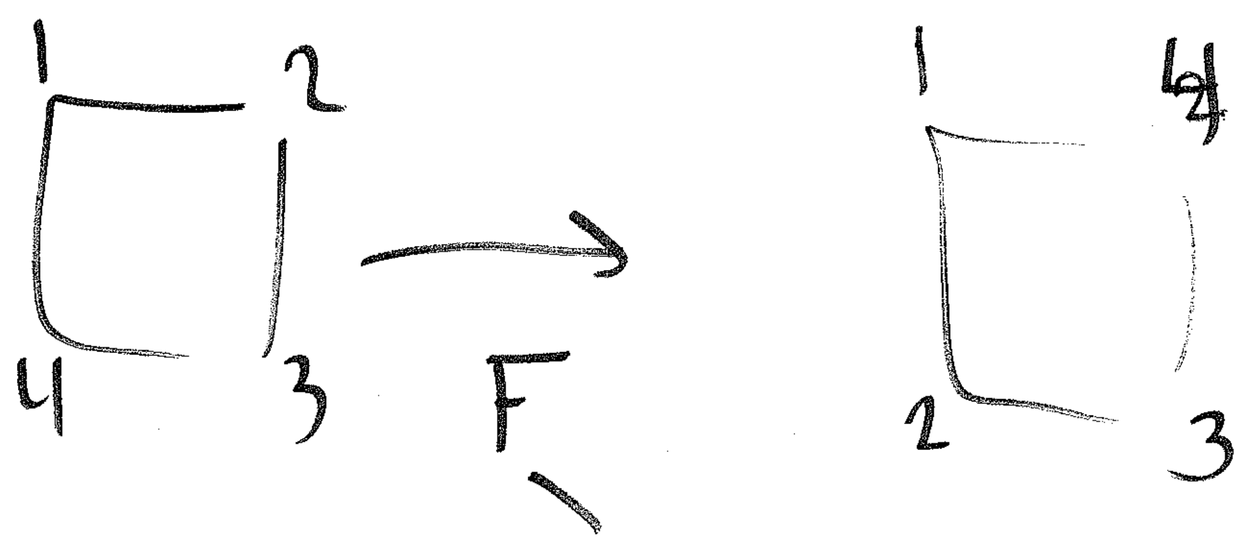
$$R_{270} = (1,4,3,2) \qquad R_{90}^{3} = R_{270}$$

$$= (4,3,2,1)$$

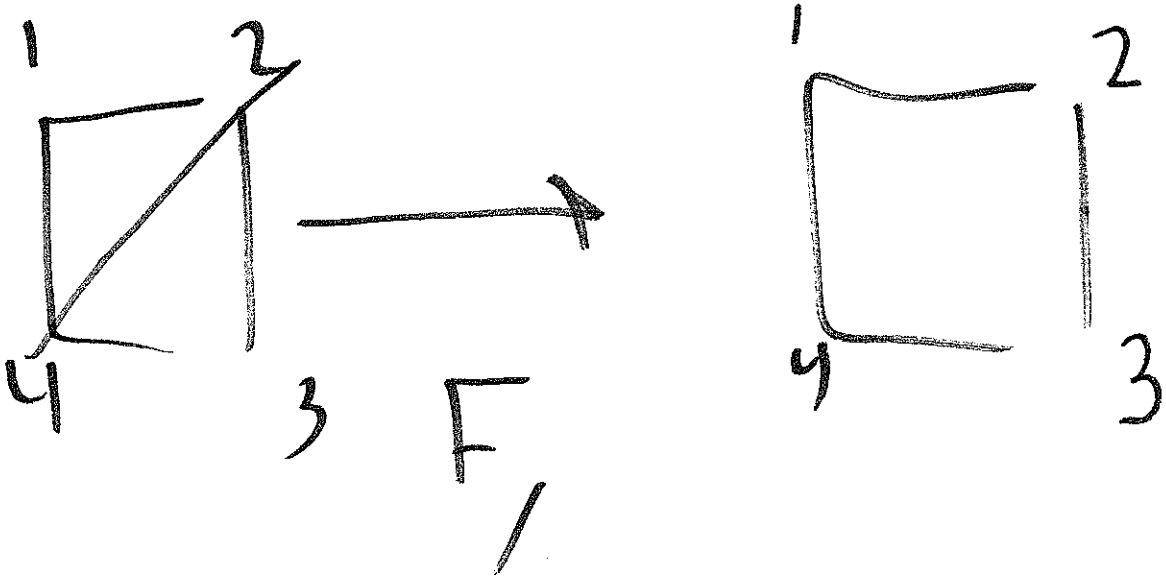


$$R_{180^\circ} = (1\,3)(2\,4)$$

$$R_{0^\circ} = R_{360^\circ}$$

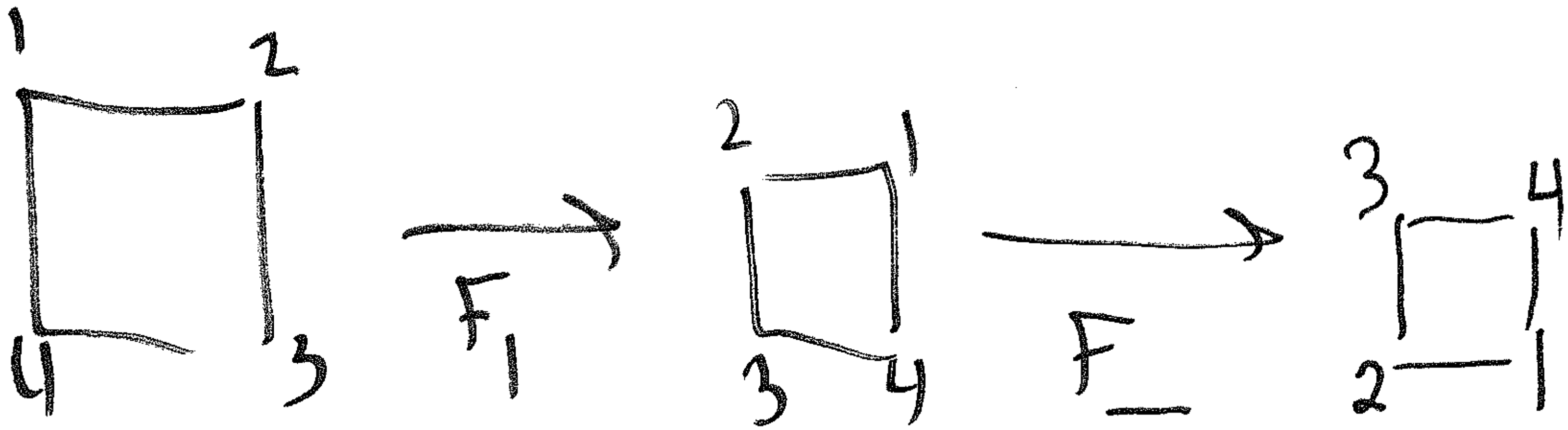


$$F_{\backslash} = (2,4)$$



$F_/ = (1,3)$

$$F_{-}F_{1} = (13)(24) = R_{180^\circ}$$



$$(14)(23)\,(12)(34) = (13)(24)$$

$\text{Flip} \circ \text{Flip} = \text{Rotation}$

$\text{Flip} \circ \text{Rotation} = \text{Flip}$

$\text{Rotation} \circ \text{Rotation} = \text{Rotation}$