Rocky Mountain Discrete Math Days  
October 21-22, 2011  
University of Wyoming

All talks are in room 121 of the Business building

Friday’s Schedule

12:30-12:50  Registration, Business building room 121
12:50- 1:00  Opening remarks
1:00- 1:30  Anton Betten  
Graph theoretic methods for computational finite geometry
1:30- 2:00  Andrew Bondi
2:00- 2:30  Colin Garnett  
The nilpotent-centralizer method & spectrally arbitrary patterns
2:30- 3:00  Jenya Kirshtein  
Cayley-Dickson loops
3:00- 3:30  Break
3:30- 4:30  Winnie Li  
Various aspects of Ramanujan graphs
4:30- 5:00  Paul Horn  
Density jumps in multigraphs
6:00- Conference dinner at the Altitude Chophouse & Brewery  
320 South 2nd Street
Saturday’s Schedule

9:30-10:00 Jason Williford
Graphs derived from generalized Kac-Moody algebras

10:00-10:30 Eric Nelson
Delsarte and strongly regular graphs

10:30-11:00 Break

11:00-11:30 Reshmi Nair
Acyclic matrices with the extreme number of distinct eigenvalues.

11:30-12:00 Keivan Hassani Monafred
The permanent rank

12:00-1:30 Lunch, Pizza provided

1:30-2:30 Chris Hall
Applications to arithmetic geometry

2:30-3:00 Seyfill Turkelli
On the gonality of modular curves in positive characteristic

3:00-3:30 Break

3:30-4:00 Sebi Cioaba
Eigenvalues, matchings and expanders

4:00-4:30 Petr Vojtechovsky
Incidence properties of cosets

4:30-5:00 Michael Kinyon
Loops with exponent 3 in all isotopes and latin squares
Anton Betten  
Colorado State University  

Graph theoretic methods for computational finite geometry

Finite Geometry leads us to many interesting but often difficult problems, which are amenable to computer search. The symmetry of the space means that many objects are isomorphic. In order to classify a set of objects, the method of breaking the symmetry can be used. Once the symmetry is broken, down-to-earth graph theory comes in. Essentially, the original hard problem is converted into a different, probably equally hard problem, namely that of finding cliques in colored graphs. Sometimes, linear programming can be used. Some specific problems that fall into this category are the classification of BLT-sets and of translation planes. During a recent trip to Germany, the speaker was surprised to see that the same algorithmic method applies to problems of such different nature.

Andrew Bondi  
Colorado State University

Colin Garnett  
University of Wyoming

The nilpotent-centralizer method & spectrally arbitrary patterns

This talk discusses a new criterion for an $n \times n$ sign-pattern, $A$, to be spectrally arbitrary: that is to have the property that for each monic real polynomial $r(x)$ of degree $n$ there exists a matrix with sign pattern $A$ that has $r(x)$ as its characteristic polynomial. Some of the implications of this new method will also be discussed.
Jenya Kirshtein  
University of Denver  

Cayley-Dickson loops  

The Cayley-Dickson loop $Q_n$ is the multiplicative closure of basic elements of the algebra constructed by $n$ applications of the Cayley-Dickson doubling process (the first few examples of such algebras are real numbers, complex numbers, quaternions, octonions, sedenions). We discuss some properties of the Cayley-Dickson loops, and describe the structure of their automorphism groups, multiplication groups, and inner mapping group.

Winnie Li  
Pennsylvania State University  

Various aspects of Ramanujan graphs  

Abstract: Briefly speaking, expanders are efficient communication networks, and Ramanujan graphs are optimal expanders. The purpose of this talk is to use Ramanujan graphs as a theme to showcase the interplay between combinatorics and number theory. Higher dimensional analogues of Ramanujan graphs will also be discussed.

Paul Horn  
Harvard University  

Density Jumps in Multigraphs  

A corollary of the Erdős-Stone theorem is that, for any $0 \leq \alpha < 1$, graphs with density greater than $\alpha$ contain an (arbitrarily) large subgraph of density at least $\alpha + c$ for some fixed $c = c(\alpha)$, so long as the graph itself is sufficiently large. This phenomenon is known as a jump at $\alpha$. Erdős conjectured that similar statements should hold for hypergraphs, and multigraphs where each edge can appear with multiplicity at most $q$, for $q \geq 2$ fixed. Brown, Erdős, and Simonovits answered this conjecture in the affirmative for $q = 2$, that is for multigraphs where each edge can appear at most twice. Rödl answered the question in the negative for hypergraphs, and later Rödl and Sidorenko answered the question in the negative for multigraphs where $q \geq 4$. No jumps or non-jumps were known for $q = 3$. In this talk we investigate some related questions. In particular, we exhibit the first known jumps for multigraphs where $q = 3$. We also exhibit a new proof of the theorem of Rödl and Sidorenko that there are non-jumps for multigraphs with $q \geq 4$. This proof uses tools from spectral graph theory, and allows us to exhibit previously unknown non-jumps. This is based on joint work with V. Rödl and S. LaFleur.
Jason Williford  
University of Wyoming  

Graphs derived from generalized Kac-Moody algebras

In this talk, we will discuss a family of graphs related to the high girth graphs $D(k,q)$. The graphs $D(k,q)$ were originally constructed by utilizing a bilinear product based on the root system of an affine Lie algebra. We give a modification of this construction which applies to Generalized Kac-Moody algebras of rank 2. Using these constructions we obtain a new lower bound on the maximum number of edges in graphs without 14-cycles. This is joint work with Art Terlep.

Eric Nelson  
Colorado State University  

Delsarte and strongly regular graphs

In 1972 Ph. Delsarte wrote a paper entitled Weights of linear codes and strongly regular normed spaces. Contained within this paper is a correspondence between projective two-weight codes and strongly regular graphs. This relationship can be used to provide non-existence results in the related area of sets of type $(m,n)$ in projective spaces. A survey of the correspondence and non-existence results will be presented.

Reshmi Nair  
University of Wyoming  

Acyclic matrices with the extreme number of distinct eigenvalues

The study of structured matrices has been of interest in Combinatorial matrix theory. In particular, there has been interest in the study of the spectral properties of $S(T)$, the set of all $n$ by $n$ symmetric matrices corresponding to a tree $T$ on $n$ vertices where $a_{ij} \neq 0$ for $i \neq j$ if and only if $ij$ is an edge in $T$. We study the problem of characterizing acyclic matrices, the matrices whose nonzero entries are described by a tree $T$. We will introduce a new technique based on Smith Normal Form and Hamming distance. Using this technique, we characterize the acyclic matrices that have at most 5 distinct eigenvalues, and give some results for acyclic matrices with larger diameter. We also characterize the acyclic matrices that have at most 2 multiple eigenvalues and whose sum of multiplicities is the maximum possible.
Keivan Hassani Monfared  
University of Wyoming  
The permanent-rank

The permanent of an \( n \times n \) matrix \( A = [a_{ij}] \) is defined to be the sum of all diagonal products of \( A \). The term-rank of \( A \), \( \text{tr}(A) \), is the largest number of nonzero entries of \( A \) with no two in the same row or column. The permanent-rank of \( A \), \( \text{perrank}(A) \), is the largest size of a square submatrix of \( A \) with nonzero permanent. Here we propose and study the following conjecture: For any real matrix \( A \), \( \text{perrank}(A) \geq \lceil \text{tr}(A)/2 \rceil \).

Chris Hall  
University of Wyoming  
Applications to arithmetic geometry

Seyfi Turkelli  
University of Georgia-Athens  
On the gonality of modular curves in positive characteristic

In this talk, I will discuss the gonality of modular curves in positive characteristic. More precisely, I will prove that, under some conditions, the gonality of modular curves of D-elliptic sheaves tends to infinity as the degree of the level structure tends to infinity.

Sebi Cioaba  
University of Delaware  
Eigenvalues, matchings and expanders

In this talk, I will describe a sufficient and best possible eigenvalue condition that implies a lower bound on the matching number of a regular graph. I will also show how one can construct new expanders by perturbing old expanders using perfect matchings.
Petr Vojtechovsky  
University of Denver  
Incidence properties of cosets

The Lagrange Theorem states that the order of a subgroup divides the order of a group by showing that two left cosets are either disjoint or they coincide. In this talk we will investigate what happens when the assumption of associativity in groups is replaced with the weaker assumption that the multiplication table is a latin square. The incidence properties of cosets are then very interesting. For instance, the blocks of any symmetric design can be realized as cosets. This is joint work with M. Kinyon and K. Pula.

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Michael Kinyon  
University of Denver  
Loops with exponent 3 in all isotopes and latin squares

It was shown by van Rees that a latin square of order $n$ cannot have more than $n^2(n - 1)/18$ latin subsquares of order 3. He conjectured that this bound is only achieved if $n$ is a power of 3. There is a mysterious family of Steiner triple systems associated to any van Rees latin square, and using these, we show that a necessary condition for such a square to exist is that $n \equiv 3 \mod 6$. There are several conditions that are equivalent to achieving the van Rees bound, some of which are algebraic. One is that the Cayley table of a loop achieves the van Rees bound if and only if every loop isotope has exponent 3. Such loops, which we call van Rees loops, form an equational class, that is, a variety. The van Rees conjecture holds for some classical varieties of loops, such as Moufang, Bol or conjugacy closed loops. In a van Rees loop, any subloop of index 3 is normal and there are exactly 6 nonassociative van Rees loops of order 27 with a non-trivial nucleus. This is joint work with Ian Wanless (Monash Univ.)