Stationary Discontinuities in Hysteretic CO_2/brine Two-phase Flow

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Abstract

Relative permeability depends on wettabitlity properties of the fluids, the displacement process, such as drainage or imbibition, and the displacement history. Hysteresis in relative permeability has a strong influence on the ability of injected and resident fluids to flow. This suggests that we have to consider relative permeability hysteresis in CO_2sequestration models. Here, we construct a mathematical model to accommodate the drainage, imbibition and scanning cycles measured in laboratory experiments of two-phase CO_2/brine flow.

The system of partial differential equations for this model is fully nonlinear. Ignoring the effects of capillary pressure, we solve the associated Riemann problem analytically and with state-of-the-art numerical methods. The analytic solution of the Riemann problem is used to verify the numerical simulations.

The Riemann solutions generically display stationary discontinuities, which might provide a mechanism for CO_2trapping. The combined mathematical and computational investigation of two-phase CO_2/brine flow suggests that the proposed hysteresis model provides a sound foundation for scientifically correct simulations of the critical mechanism for CO_2sequestration due to gas trapping as a result of permeability hysteresis.

1. Introduction

In this paper, we construct a mathematical model to accommodate the drainage, imbibition and scanning cycles measured in laboratory experiments of two-phase

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CO_2/brine flow. The system of partial differential equations for this model is fully nonlinear. Ignoring the diffusive effects of capillary pressure, we solve the associated Riemann problem analytically and with state-of-the-art numerical methods. The analytic solution of the Riemann problem is used to verify the numerical simulations.

Mathematically, hysteresis effects are modeled by assuming that there are two irreversible permeability curves for CO_2, the imbibition and drainage curves, and there is a continuous family of reversible curves, called scanning curves. These assumptions are motivated by the experimental work for two-phase CO_2/brine flow performed at deep reservoir conditions [1]. We refer the reader for the works on effect of hysteresis on the multiphase flow in porous media to [3, 6, 2].

In this paper, we construct a mathematical model to accommodate the drainage, imbibition and scanning cycles measured in laboratory experiments of two-phase CO_2/brine flow. We consider the injection problem for a model of two-phase CO_2/brine flow in a core sample of porous rock, taking into account hysteresis effects in the permeability of the CO_2 phase. Ignoring the diffusive effects of capillary pressure, we solve the associated problem with state-of-the-art numerical methods. Since the analytic solution for the Riemann problem in one-dimensional framework for the proposed model can be constructed, the analytic solution is used to verify the numerical solutions.

This paper is organized as follows. In section 2 we describe the hysteresis mathematical model for CO_2/brine two-phase flow. In section 3 we describe the numerical solution for the system of partial differential equations for the proposed mathematical model. The Kurganov-Tadmor [5] type central scheme is used to solve the brine saturation equation. In section 4 numerical results in one-dimensional setup are presented and verified using analytic solution of the Riemann problem. Section 5 contains the conclusion and future work.

2. Hysteresis Model

In this section, we describe a simplified mathematical model of two-phase imbibition relative permeability hysteresis and non-wetting phase trapping. This model is qualitatively consistent with the laboratory experiments of two-phase CO_2/brine flow performed at deep reservoir conditions [1].

We neglect the (weak) hysteresis effects in the brine (the wetting phase) and assume that only the CO_2 phase (the non-wetting phase) exhibits relative permeability hysteresis.

During CO_2 injection (primary drainage flow), the relative permeability of CO_2 increases and the relative permeability of brine decreases. Thus, we assume

Figure 1: Description of the model. Left: Relative permeability curves. Center: Domain of scanning curves. Right: Fractional flow curves.

the drainage CO_2 phase relative permeability is

$$k^d_{\mathrm{CO}_2}(s) = (1-s)^2, \tag{2.1}$$

and the brine relative permeability function is

$$k_{Br}(s) = s^2, \tag{2.2}$$

where s is the saturation of brine.

If a flow reversal occurs during the primary drainage flow (i.e., if brine is injected), the resulting flow will be called a *scanning flow*. During a scanning flow the CO_2 relative permeability does not decrease along the drainage path, but rather decreases faster, becoming zero at a positive CO_2 saturation (see Figure 1). The latter saturation is an *irreducible saturation* and it is a function of the CO_2 saturation at flow reversal. Denoting the brine saturation at flow reversal by π, we then assume the following model for the CO_2 relative permeability during scanning flow:

$$k_{\mathrm{CO}_2}(s,\pi) = (1-\pi)(1+\pi-2s). \tag{2.3}$$

Accordingly, the domain of the scanning relative permeability functions consists of the saturation values in the interval $\pi \leq s \leq (1+\pi)/2$.

To conclude the description of our simplified hysteresis model we assume the changes in permeability during scanning flow are reversible.

3. Numerical Model

In this section, we describe the numerical solution of the system of partial differential equations, namely, the saturation equation

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x}F(s,\pi) = 0, x \in \mathbf{R}, \tag{3.1}$$

where

$$F(s,\pi) = \begin{cases} \frac{s^2\{\nu+\epsilon(1-\pi)(1+\pi-2s)\}}{s^2+\mu(1-\pi)(1+\pi-2s)} & \text{for scanning flows,} \\ \frac{s^2\{\nu+\epsilon(1-s)^2\}}{s^2+\mu(1-s)^2} & \text{for drainage.} \end{cases} \tag{3.2}$$

Here we define $\mu = \mu_{Br}/\mu_{\mathrm{CO}_2}$, and ν and ϵ are real parameters. When $\nu = 1$ and $\epsilon = 0$, we get the pure convection case. For $\nu = 0$ and $\epsilon = 1$, we get the system with pure gravitational effect.

3.1. Numerical Schemes

In this subsection, we present the high-resolution central-upwind central schemes for solving (3.1) [4]. This Godunov-type semi-discrete central schemes are based on the use of more precise information about the local speeds of propagation and can be viewed as a generalization of Kurganov-Tadmor central schemes [5]. The main advantages of the proposed central schemes are the high resolution, due to the smaller amount of the numerical dissipation, and the simplicity. There are no Riemann solvers and characteristic decomposition involved. At the same time, the central-upwind schemes have an upwind nature, since they respect the directions of wave propagation by measuring the one-sided local speeds.

Define $x_i := i\Delta x$, $x_{i\pm\frac{1}{2}} := x_i \pm \frac{\Delta x}{2}$. Assume that we have already computed an approximation to the solution at time level $t = t^n$ of the form,

$$\tilde{s}(x,t^n) := \sum_i p_i^n(x)\chi_i(x), \tag{3.3}$$

where

$$p_i(x) = \bar{s}_i + (s_x)_i(x - x_i), \tag{3.4}$$

with slopes approximated by MinMod limiter, χ_i is the characteristic function of the corresponding region, and $\bar{s}_i$ is the cell average defined by

$$\bar{s}_i := \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} p_i^n(x)\,dx. \tag{3.5}$$

We denote the cell interface value at the faces

$$s^+_{i+\frac{1}{2}} := p^n_{i+1}(x_{i+\frac{1}{2}}),\ s^-_{i+\frac{1}{2}} := p^n_i(x_{i+\frac{1}{2}}) \tag{3.6}$$

We then compute the maximum and minimum local speeds of propagation of the discontinuities by

$$
\begin{aligned}
a^+_{i+\frac{1}{2}} &:= \max_{s\in\mathcal{C}\left(s^+_{i+\frac{1}{2}},s^-_{i+\frac{1}{2}}\right),\pi\in(\pi_i,\pi_{i+1})} \left\{\frac{\partial F}{\partial s}(s,\pi),0\right\},\\
a^-_{i+\frac{1}{2}} &:= \min_{s\in\mathcal{C}\left(s^+_{i+\frac{1}{2}},s^-_{i+\frac{1}{2}}\right),\pi\in(\pi_i,\pi_{i+1})} \left\{\frac{\partial F}{\partial s}(s,\pi),0\right\}.
\end{aligned}
\tag{3.7}
$$

Now we can write the semi-discrete central-upwind scheme as

$$
\frac{d}{dt}\bar{s}_i(t) = -\frac{H_{i+\frac{1}{2}}(t) - H_{i-\frac{1}{2}}(t)}{\Delta x} \tag{3.8}
$$

with the numerical flux

$$
H_{i+\frac{1}{2}}(t) := \frac{a^+_{i+\frac{1}{2}}F(s^+_{i+\frac{1}{2}},\pi_{i+1}) - a^-_{i+\frac{1}{2}}F(s^-_{i+\frac{1}{2}},\pi_i)}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}} + \frac{a^+_{i+\frac{1}{2}}a^-_{i+\frac{1}{2}}}{a^+_{i+\frac{1}{2}} - a^-_{i+\frac{1}{2}}}\left[s^+_{i+\frac{1}{2}} - s^-_{i+\frac{1}{2}}\right]. \tag{3.9}
$$

3.2. Time marching for the semi-discrete central schemes

We can use SSP Runge-Kutta scheme [8] for the time integration in solving equation (3.8). However, in some cases the time step can be very small due to their strict stability region. Therefore, we use the explicit embedded integration, third-order RK method introduced by Medovikov [7] (his original code, DUMKA3,was used).

4. Numerical Results

In this section, we present the numerical results to demonstrate the effect of hysteresis in two-phase CO_2/brine flow. We consider a one-dimensional domain of unit length. Let $\mu = \mu_{Br}/\mu_{CO_2} = 10$. Initially, for $x < 0.5$ we have the brine saturation as 0.9, and for $x > 0.5$ we have the brine saturation as 0.5. Figure 2 shows the results for the convection-dominated case, where $\nu = -1.0$ and $\epsilon = 0.1$ in (3.2), at $t = 0.05$, and for the gravity-dominated case, where $\nu = -0.1$ and $\epsilon = 1.0$ in (3.2), at $t = 0.5$.

In Figure 2, we consider the convection-dominated case. With the hysteresis effect, we can observe a stationary discontinuity present at the interface. When the fractional flow curve of brine jumps horizontally from the scanning curve $\pi = 0.8$ to the drainage, we have the stationary discontinuity (see the fractional flow curve in red). This stationary discontinuity helps to capture CO_2 for much longer time in the system. A similar argument follows for the gravity-dominated case.

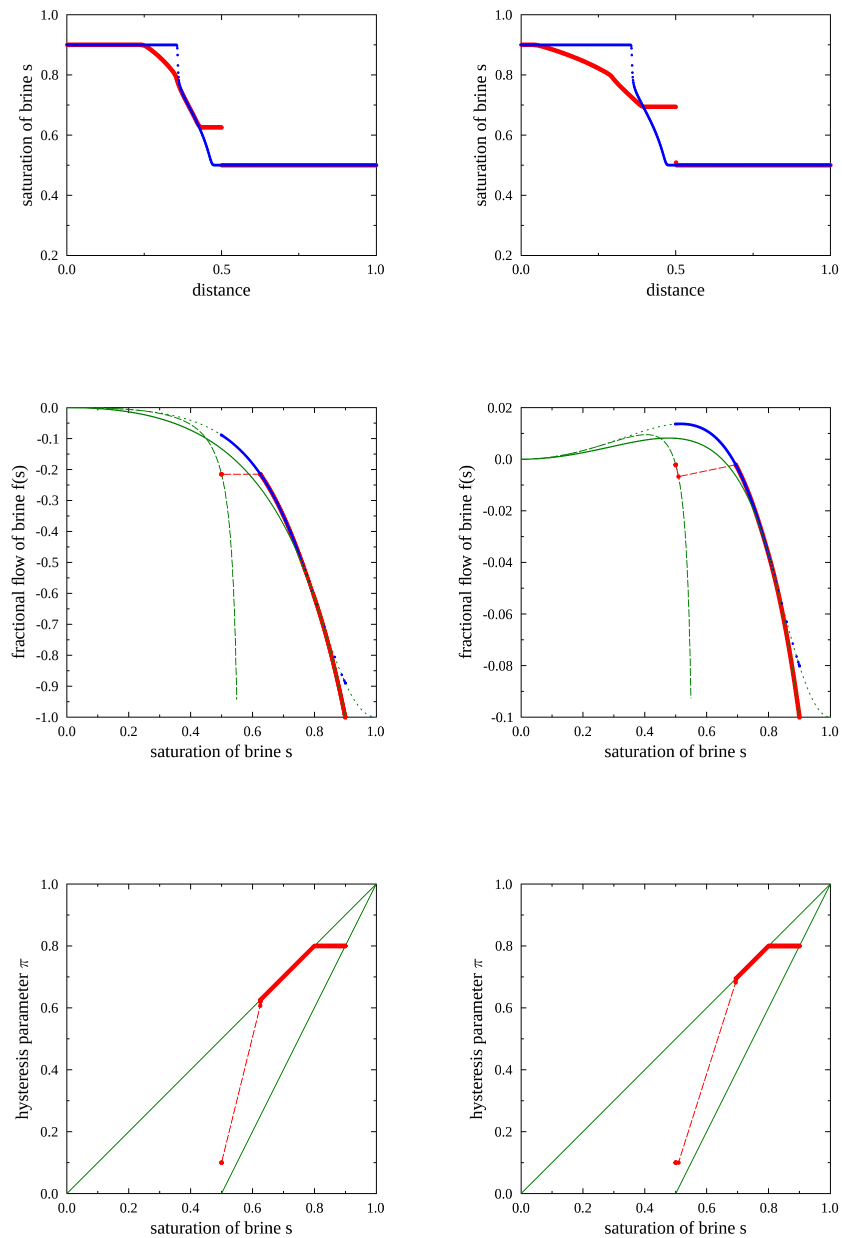

Figure 2: Left: Convection-dominated case at $t = 0.05$. Right: Gravity-dominated case at $t = 0.5$. The red line denotes the case with the hysteresis effect and the blue line denotes the case without the hysteresis effect.

5. Conclusion

In this paper, we constructed a mathematical model to accommodate the drainage, imbibition and scanning cycles measured in laboratory experiments of two-phase CO_2/brine flow. The analytic solution of the Riemann problem is used to verify the numerical simulations. From the numerical solutions, we have shown that the stationary discontinuities in hysteretic CO_2/brine two-phase flow help to capture CO_2 for much longer time. These discontinuities result from the assumed reversibility of the scanning relative permeability functions. We suggest that additional experimental investigation should be performed to verify their existence (or not) in real flows.

6. References

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