Homework 1 (MATH 2310-04) **Name (Print):**

Due date: Thursday, Feb. 6, 2014

1. Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$:

$$t\frac{dy}{dt} + 2y = \sin(t), \quad t > 0$$

Solution: $y(t) = -\frac{\cos(t)}{t} + \frac{\sin(t)}{t^2} + \frac{C}{t^2} \quad \to \quad y = 0 \quad \text{for } t \to \infty$

2. Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$:

$$\frac{dy}{dt} + 2ty = 2te^{-t^2}$$

Solution: $y(t) = t^2 e^{-t^2} + Ce^{-t^2} \quad \to \quad y = 0 \quad \text{for } t \to \infty$

3. Consider an insulated box (a building, perhaps) with internal temperature u(t). According to Newton's law of cooling, u(t) satisfies the differential equation

$$\frac{du}{dt} = -k[u - T(t)],$$

where T(t) is the ambient (external) temperature. Suppose that T(t) varies sinusoidally,

$$T(t) = T_0 + T_1\cos(\omega t).$$

Here, k, ω, T_0 and T_1 are positive constants.

a) Find the temperature u at any time t.

b) Find the temperature u as t becomes large.

c) Suppose that $\omega = 0$. Find the temperature u as t becomes large. Explain your result by taking reference to the differential equation for u.

The following integral will be helpful (a and b may be any constants):

$$\int e^{ax}\cos(bx)dx = \frac{e^{ax}}{a^2 + b^2}[a\cos(bx) + b\sin(bx)]$$

Solution:

a) $u(t) = T_0 + \frac{kT_1}{k^2 + \omega^2}[k\cos(\omega t) + \omega\sin(\omega t)] + Ce^{-kt}$

b) $u(t) = T_0 + \frac{kT_1}{k^2 + \omega^2}[k\cos(\omega t) + \omega\sin(\omega t)]$

c) $u(t) = T_0 + T_1$