Homework 2 (MATH 2310-04) **Name (Print):**

Due date: Thursday, Feb. 13, 2014

1. Solve the given differential equations:

   a) $\dfrac{dy}{dx} + y^2 \sin(x) = 0$

   b) $\dfrac{dy}{dx} = \dfrac{x^2}{y(1+x^3)}$

   Solution: a) $y(x) = 1/(C - \cos(x))$ b) $y(x) = \pm(2\ln|1+x^3|/3 + C)^{1/2}$

2. Solve the following initial value problem and determine where the solution attains its maximum value.

   $\dfrac{dy}{dx} = \dfrac{2-e^x}{3+2y}$ $y(0) = 0$ **Solution :** $y(x) = -3/2 + (2x - e^x + 13/4)^{1/2}$

   The solution attains a global maximum at $x = \ln 2$.

3. A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ g/liter of salt enters the tank at a rate of 2 liters /min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

   Solution : Differential equation : $\dfrac{dm}{dt} = 2\gamma - \dfrac{m}{60}$

   Solution : $m(t) = 120\gamma(1 - e^{-t/60}) \to \quad m = 120\gamma \quad \text{for } t \to \infty$