Homework 5 (MATH 5400-01) **Name (Print):**

Due date: Friday, Oct. 12, 2012

1. Solve formally by separation of variables:

$$u_{tt} = c^2 \Delta u, \qquad 0 < x < a, 0 < y < b, t > 0$$
$$u(x, y, 0) = f(x, y), \quad u_t(x, y, 0) = g(x, y), \quad 0 \le x \le a, 0 \le y \le b$$
$$u_y(x, 0, t) = 0, \quad u_y(x, b, t) = 0, \quad 0 \le x \le a, t \ge 0$$
$$u(0, y, t) = 0, \quad u(a, y, t) = 0, \quad 0 \le y \le b, t \ge 0$$

2. Solve formally by separation of variables:

$$u_{tt} = c^2 \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) + f(r, t), \qquad 0 \le r < a, t > 0$$
$$u(r, 0) = 0, \quad u_t(r, 0) = 0, \qquad 0 \le r \le a$$
$$u(a, t) = 0, \qquad t \ge 0$$

Hint: Let $R_n(r)$ be the eigenfunctions of the homogeneous PDE (for f(r, t) = 0). Assume a solution of the form

$$u(r, t) = \sum_{n=1}^{\infty} \phi_n(t) R_n(r).$$

Expand

$$f(r, t) = \sum_{n=1}^{\infty} f_n(t) R_n(r).$$