

Homework 8 (MATH 5400-01)**Name (Print):****Due date: Monday, Nov. 26, 2012**

1. Solve this problem by means of an eigenfunction expansion:

$$y'' + 2y = -x, \quad y(0) = 0, \quad y(1) = 0$$

- a) Find the normalized eigenfunctions of the homogeneous problem $y'' + \lambda y = 0$.
b) Follow the development of Sect. 6.4 to find $y(x)$.

Hint: Be careful regarding the sign of $f(x)$!

2. Find the extremal of the functional

$$I(y) = \int_0^1 (y'^2 + 3y + 2x) dx, \quad y(0) = 0, \quad y(1) = 1.$$

3. Consider a simple pendulum of length L and bob mass m suspended from a frictionless support. To describe the state at any time t we choose the coordinate θ measuring the angle displaced from the vertical position. The kinetic energy T and potential energy V are given by

$$T = \frac{m}{2} L^2 \dot{\theta}^2, \quad V = mgL(1 - \cos \theta).$$

Use Hamilton's principle to find the evolution equation for this system.