Homework 2 (MATH 4300-01) **Name (Print):**
Due date: Friday, Sept. 20, 2013

1. The following data describe the changes of a certain population P in time t (in days). Develop a model for the data based on the logistic model $P = a / [1 + b\, e^{ct})]$.

t	1	2	3	4	5	6	7
P	43	58	76	90	97	106	112

a) Rewrite this function as a linear relation between redefined variables.
b) Graph the data such that the linearity assumption can be tested. Compare the data in this plot with a linear function to find the model parameters.
c) Present the model obtained and graph both the model and the original data given in the table. Graph the relative error of the model.
d) Use the model to predict the time at which the initial population at $t = 0$ is increased by a factor of 2.

2. Consider the development of the world population in time from 1804–2050 according to the Decennial Censuses, U.S. Census Bureau, U.S. Dept. of Commerce (World Almanac 2010). The population P is measured in 10^9 and t refers to the year. The last two population values are projections.

t	1804	1927	1960	1974	1987	1999	2009	2025	2050
P	1.0	2.0	3.0	4.0	5.0	6.0	6.77	7.95	9.32

Assume that the population P can be described by the function

$$P = \frac{a}{1 + b e^{ct}} + d,$$

where a, b, c, and d are any constants. For a certain time period before 1804, the population can be approximated by a constant value $P = 1$. Assume that the population density levels off finally at a value of $P = 11$.
a) Rewrite the model for P as a linear relation between redefined variables.
b) Graph the data such that the linearity assumption can be tested. Compare the data in this plot with a linear function to find the model parameters.
c) Present the model obtained and graph both the model and the original data given in the table. Graph the relative error of the model.
d) Find the time at which the population change dP / dt has a maximum.