Homework 4 (MATH 4300-01) **Name (Print):**
Due date: Friday, Oct. 4, 2013

1. Consider the least-squares error E^2 for a linear function $y_M = a\,x + b$,

$$E^2 = \left\langle \tilde{Y}^2 \right\rangle - 2a\left\langle \tilde{X}\tilde{Y} \right\rangle + a^2\left\langle \tilde{X}^2 \right\rangle + \left(b - \langle Y \rangle + a\langle X \rangle\right)^2.$$

   a) Determine directly the optimal b value that minimizes E^2. Do not apply the Second Derivatives Test.
   b) Determine in the same way the optimal a value that minimizes E^2.

2. Assume that this data set can be modeled by a power function $y_M = a\,x^b$.

X	1	2	3	4	5
Y	0.4	0.65	0.86	1.06	1.23

   a) Introduce new variables that are linearly related. Find the optimal model parameter values for this linear model by using the least-squares error.
   b) Graph the resulting optimal power function $y_M = a\,x^b$ and the data.

3. Assume that this data set can be modeled by a power function $P = P_0 + b\,t^a$.

t	0	1	2	3	4
P	100	101	108	140	230

   a) Introduce new variables that are linearly related. Find the optimal model parameter values for this linear model by using the least-squares error.
   b) Graph the resulting optimal power function $P = P_0 + b\,t^a$ and the data.