Homework 6 (MATH 4300-01) **Name (Print):**
Due date: Friday, Nov. 1, 2013

1. The period of a damped pendulum is given by $T_P = 2\pi\, c_P\, (r / g)^{1/2}$, where the factor is given by the function $c_P = c_P(\alpha_0, \mu_*)$. The damping parameter is defined by $\mu_* = \mu\, (r^3 / g)^{1/2} / m$. Here, r is the length of the pendulum, g is the gravity acceleration, α_0 denotes the initial angle of displacement, μ is the dynamic viscosity, and m is the pendulum mass.
   a) Describe an experiment for the calculation of c_P.
   b) Derive the design conditions for a model pendulum that is 50% smaller than a real pendulum.
   c) Specify the design conditions for the case that the same fluid is used.

2. Consider this case: A windmill is rotated by air flow to produce power to pump water. You have to find the power output P (dimension $M\, L^2\, T^{-3}$) of the windmill. It can be expected that P depends on the diameter d of the windmill, the wind speed v, the air density ρ, the air viscosity μ, and the rotational speed ω (dimension T^{-1}) of the windmill.
   a) Use dimensional analysis to calculate P. Use the exponents of P, μ, and ω as independent parameters.
   b) Provide the design condition for a model that is q times smaller than the real windmill. Use the same fluid in the model system.

3. Consider a steady laminar fluid flow through a smooth horizontal pipe. The pressure drop Δp (dimension $M\, L^{-1}\, T^{-2}$) between two points along the pipe depends on the distance d between the two points, the diameter D of the pipe, the fluid density ρ, the fluid viscosity μ, and the fluid velocity v.
   a) Use dimensional analysis to calculate Δp. Use the exponents of Δp, μ, and d as independent parameters.
   b) Provide the design condition for a model that is q times smaller than the real system. Use the same fluid in the model system.