Geometric Series
\[ \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r} \text{ for } |r| < 1 \]

p-Series
\[ \sum_{n=1}^{\infty} \frac{1}{n^p} \] converges for \( p > 1 \), otherwise, diverges.

Divergence Test
If \( \lim_{n \to \infty} a_n \neq 0 \) or does not exist, then \( \sum_{n=1}^{\infty} a_n \) diverges.

Note: This does not mean \( \lim_{n \to \infty} a_n = 0 \) implies \( \sum_{n=1}^{\infty} a_n \) converges.

Integral Test
If \( f \) is a continuous, positive, decreasing function on \([1, \infty)\) and \( f(n) = a_n \), then
if \( \int_{1}^{\infty} f(x) \) diverges, so does \( \sum_{n=1}^{\infty} a_n \),
if \( \int_{1}^{\infty} f(x) \) converges, so does \( \sum_{n=1}^{\infty} a_n \).

Comparison Test
Suppose \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) have postive terms. Then,
if \( \sum_{n=1}^{\infty} b_n \) diverges and \( b_n \leq a_n \forall n \), then \( \sum_{n=1}^{\infty} a_n \) diverges,
if \( \sum_{n=1}^{\infty} b_n \) converges and \( b_n \geq a_n \forall n \), then \( \sum_{n=1}^{\infty} a_n \) converges.

Limit Comparison Test
Suppose \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) have postive terms. Then, if \( \lim_{n \to \infty} \frac{a_n}{b_n} = c > 0 \) and \( c \) finite, both \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) converge or both diverge.

Alternating Series Test
If the alternating series \( \sum_{n=1}^{\infty} (-1)^{n-1} b_n \) satisfies
1. \( b_{n+1} \leq b_n \)
2. \( \lim_{n \to \infty} b_n = 0 \)
then the series converges.

Absolute Convergence
A series \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent if the series \( \sum_{n=1}^{\infty} |a_n| \) converges.
If a series is absolutely convergent, then it is convergent.
Note that a convergent series may not be absolutely convergent.

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1 J. Prewett, 2009
Ratio Test

1. If \( \lim_{n \to \infty} \frac{|a_{n+1}|}{a_n} = L < 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) absolutely converges.

2. If \( \lim_{n \to \infty} \frac{|a_{n+1}|}{a_n} = L > 1 \) or \( = \infty \), then the series \( \sum_{n=1}^{\infty} a_n \) diverges.

Note that if \( L = 1 \), we get no information from the Ratio Test.

Root Test

1. If \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L < 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is absolutely convergent.

2. If \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L > 1 \), then the series \( \sum_{n=1}^{\infty} a_n \) is divergent.

3. If \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L = 1 \), then the test is inconclusive.

Power Series

For what values of \( x \) is the series convergent? Use Ratio Test.

Find radius and interval of convergence.

Represent a function such as \( \frac{1}{x + 2} \) as a power series.

Taylor and Maclaurin Series

Taylor Series about \( a \): \( f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \).

Maclaurin Series: \( f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \).

You should know or write down the Taylor Series for functions in the box on page 618. (Skip the one for \( \tan^{-1}x \))

Write a function as a series to integrate it.

Taylor’s Inequality to estimate the error of the \( n^{th} \) degree Taylor Polynomial.

\[^2\text{J. Prewett, 2009}\]