Q-polynomial regular near 2d-gons

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Near polygons were introduced by Shult and Yanushka [3].
A near 2d-gon, d ≥ 2, is a connected graph of diameter d such that:

1. there are no induced subgraphs of shape K_{1,1,2}:

2. for every vertex p and every maximal clique ℓ, there is a unique vertex p’ in ℓ at minimal distance from p in the graph.

Note that by the first condition, any two adjacent vertices are in a unique maximal clique. Maximal cliques in near 2d-gons are referred to as lines.

The near 2d-gons with exactly two vertices in each line are precisely the bipartite graphs of diameter d. Near 4-gons, 6-gons and 8-gons are also referred to as near quadrangles, hexagons and octagons, respectively.

A near 2d-gon is regular if it is distance-regular: when |Γ_i(x) ∩ Γ_j(y)| only depends on i, j and d(x,y). (Much more on distance-regular graphs in general can be found in [2].) In that case, it is of a certain order (s, t_2, . . . , t_d = t).

Bannai and Ito [1] proposed the problem of classifying all distance-regular graphs with the so-called Q-polynomial property, which informally means that the eigenspaces of the adjacency matrix of the graph can be ordered in a “nice way”. In this talk, our goal is to discuss regular near 2d-gons with the Q-polynomial property. For regular near 2d-gons, this property only depends on the parameters (s, t_2, . . . , t_d = t).

For Q-polynomial regular near hexagons of order (s, t), s ≥ 2, the only open case is when the parameters s, t_2, t satisfy: t + 1 = (s^2 - s + 1)(t_2 + s + 1) (i.e. equality holds in the so-called Mathon bound). We will show that here (s, t_2, t) cannot be (3, 1, 34), (8, 4, 740), (92, 64, 1314560), (95, 19, 1027064) or (105, 147, 2763012).

We will also show that there are no regular near 2d-gons with parameters s, t and t_i satisfying d ≥ 4, s ≥ 2 and t_i = s^3 · \frac{(s^i - 2 - (-1)^i)(s^{i-1} + (-1)^i)}{(s^i - 1)(s + 1)} for every i ∈ {0, 1, . . . , d}. This completes the classification for Q-polynomial regular near 2d-gons of order (s, t), with d ≥ 4 and s ≥ 2.

If time permits, we will also discuss how the Q-polynomial property can be used to discuss substructures in (well-known or hypothetical) regular near 2d-gons.
References

