Math 5345 — HW # 4

Due December 15, 2005

1. Consider Burgers' equation with data

$$\overset{\circ}{u}(x) = \begin{cases} 2 & \text{if } 0 < x < 1.5, \\ 1 & \text{if } 1.5 < x < 2.5, \\ 0 & \text{otherwise.} \end{cases}$$

   Determine the entropy-satisfying weak solution for all $t > 0$. Sketch the characteristics and shock curves in the $x - t$ plane.

   Hint: Once the rarefaction and shock begin to interact, use the fact that the integral of u is conserved to determine the shock location. Then verify that the resulting shock speed satisfies the Rankine-Hugoniot condition at all times.

2. Consider the scalar conservation law

$$q_t + (q^3)_x = 0$$

   with initial data

$$\overset{\circ}{q}(x) = \begin{cases} 2 & \text{if } 0 < x < 10, \\ 1 & \text{otherwise.} \end{cases}$$

   Determine the true solution for $0 \leq t \leq 1$. (Note: the function q^3 is convex for $1 \leq q \leq 2$.)

3. Do Exercise 11.1 in the textbook. Using this result to determine the breaking time for the traffic flow problem illustrated in Figure 11.1(b), where

$$\overset{\circ}{q}(x) = 0.25 + 0.7 \exp(-x^2/100).$$

4. Do Exercise 11.2.