1. (11 points) Consider the function \( f(x) = 2^x + x. \)
   
   (a) Using your calculator, complete the following table of values for \( f \), correct to four decimal places:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
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<tr>
<td>-0.1</td>
<td></td>
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<tr>
<td>0</td>
<td></td>
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<tr>
<td>0.1</td>
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<td>1</td>
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</tbody>
</table>

   (b) Showing all of your work, use the values in (a) to compute the average rate of change of \( f(x) \) on the interval \([-0.1, 0]\).

   (c) Showing all of your work, use the values in (a) to compute the average rate of change of \( f(x) \) on the interval \([0, 0.1]\).

   (d) Using your work above, give a reasonable estimate for the instantaneous rate of change of \( f(x) \) at 0.
2. (9 points) Consider the function whose graph is shown below. At each of the following points, is the function continuous or discontinuous? If discontinuous, is it left-continuous, right-continuous or neither?

(a) $x = 2$
(b) $x = 3$
(c) $x = 5$
(d) $x = 5.001$

For problems 3,4,5,6,7, write the letter of the correct answer in the box.

3. (3 points) Which of the following is the correct formula for the slope of the tangent line to the graph of $f$ at the point $(a, f(a))$?

A. $m_{\text{tan}} = \lim_{t \to a} \frac{f(b) - f(t)}{b - a}$
B. $m_{\text{tan}} = \lim_{t \to a} \frac{f(b) - f(t)}{b - t}$
C. $m_{\text{tan}} = \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$
D. $m_{\text{tan}} = \lim_{t \to a} \frac{f(b) - f(a)}{b - a}$

4. (3 points) How are $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$ calculated if $f(x)$ is a polynomial function?

A. $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = c$
B. $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = f(c)$
C. $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$ may not exist
D. $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = 0$

5. (3 points) The graph of which function has the line $y = -1$ as a horizontal asymptote?

A. $f(x) = e^{-x}$
B. $f(x) = \frac{-x}{1 - x}$
C. $f(x) = \frac{x}{1 - x}$
D. $f(x) = \frac{x}{1 + x}$
6. (3 points) \[ \lim_{x \to \infty} \frac{10^8x^5 + 10^6x^4 + 10^4x^2}{10^8x^6 + 10^7x^5 - 10^3x^3} = \]

A. \( \frac{1}{10} \)  
B. 1  
C. \(-1\)  
D. 0  
E. \(-\frac{1}{10}\)

7. (3 points) If the function \( f(x) = \begin{cases} \frac{x-3}{\sqrt{x}-\sqrt{3}}, & \text{for } x \neq 3; \\ a, & \text{for } x = 3 \end{cases} \) is continuous at \( x = 3 \), then the constant \( a \) must have value

A. 0  
B. \( \sqrt{3} \)  
C. \( \frac{1}{\sqrt{3}} \)  
D. \( 2\sqrt{3} \)  
E. \( \frac{1}{2\sqrt{3}} \)

8. (15 points) Evaluate the following limits analytically and give the exact value in each case. Show all your work and express your answers in simplest form. (These are not calculator problems.)

(a) \[ \lim_{x \to \infty} \frac{\sqrt{11x^4 + x - 7}}{2x^2 + 13} \]

(b) \[ \lim_{x \to -2} \left( \frac{3x^3 - 3}{x^2 + 4} \right)^{-1/3} \]

(c) \[ \lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x} \]
9. (10 points) Let $f(x) = 2x^2 + 3$.

   (a) Use the **definition** of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ to find $f'(x)$ for the function above.

   (b) Using the result of (a), write the equation of the tangent line to the graph of $f$ at (2, 11).

10. (8 points) For nonzero $x$ values near 0, it can be shown that $1 - \frac{x^2}{6} \leq \frac{\sin x}{x} \leq 1$.

    (a) Use the inequality above to find $\lim_{x \to 0} \frac{\sin x}{x}$ analytically. Show your work and **do not use a table of values** to find this limit.

    (b) Which theorem covered in class allows you to evaluate the limit in part (a)?
11. **(10 points)** Consider the function \( f(x) = x^3 - x - 2. \)

(a) Using interval notation, give the values of \( x \) for which \( f(x) \) is continuous.

(b) Show that there is a number \( c \) in the interval \((1, 2)\) satisfying \( f(c) = 0 \). Name the theorem that you used for your conclusion and explain why it applies to this problem.

(c) Sketch the graph of \( f \) on the interval \([0, 3]\). Use your graph to estimate the number \( c \) satisfying \( f(c) = 0 \).

\[
\begin{array}{c|c}
 x & f(x) \\
- & - \\
0 & \\
1 & \\
2 & \\
3 & \\
\end{array}
\]

\( c \approx \)

12. **(8 points)** Using the definition of continuity, find the values of \( b \) and \( c \) that will make \( g(x) \) continuous at \( x = c \). **Solve analytically** using correct mathematical notation.

\[
g(x) = \begin{cases} 2x + b, & \text{for } x < c; \\ 14, & \text{for } x = c; \\ x^3 - 13, & \text{for } x > c \end{cases}
\]

\( b = \)

\( c = \)
13. (14 points) A continuous function $f$, whose graph is shown above, satisfies
\[
\lim_{x \to -\infty} f(x) = -\infty; \quad \lim_{x \to \infty} f(x) = 2; \quad f(-3) = 0; \quad f(0) = 3; \quad f(4) = 0.
\]

Answer the following questions about the new function $g(x) = \frac{1}{f(x)}$. (Remember that the following questions refer to $g(x) = \frac{1}{f(x)}$ rather than $f(x)$.)

(a) $g(0) = \underline{\quad}$

(b) \[\lim_{x \to -\infty} g(x) = \underline{\quad}\]

(c) \[\lim_{x \to \infty} g(x) = \underline{\quad}\]

(d) \[\lim_{x \to -3^-} g(x) = \underline{\quad}\]

(e) \[\lim_{x \to -3^+} g(x) = \underline{\quad}\]

(f) \[\lim_{x \to -3} g(x) = \underline{\quad}\]

(g) \[\lim_{x \to 4^-} g(x) = \underline{\quad}\]

(h) \[\lim_{x \to 4^+} g(x) = \underline{\quad}\]

(i) \[\lim_{x \to 4} g(x) = \underline{\quad}\]

(j) The graph of $g$ has horizontal asymptote(s) given by the equation(s): \underline{\quad}

(k) The graph of $g$ has vertical asymptote(s) given by the equation(s): \underline{\quad}

For instructors’ use only:

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