1. Sol.

\[ f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{1}{\frac{2+h+3}{h} - \frac{1}{2+3}} = \lim_{h \to 0} \frac{\frac{1}{2+h+3} - \frac{1}{2+3}}{h} \]

\[ = \lim_{h \to 0} \frac{5-(5+h)}{(5+h)5} = \lim_{h \to 0} \frac{-h}{(5+h)5} = \lim_{h \to 0} \frac{-1}{(5+h)5} \]

\[ = -\frac{1}{(5+0)5} = -\frac{1}{25} \]

2. Sol.

(a) \( f(x) = x^3 \) and \( a = 2 \).

(b) \[
\lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h} = \lim_{h \to 0} \frac{[2^3 + 3(2^2)h + 3(2)h^2 + h^3] - 2^3}{h}
\]

\[ = \lim_{h \to 0} \frac{3(2^2)h + 3(2)h^2 + h^3}{h} \]

\[ = \lim_{h \to 0} \frac{3(2^2) + 3(2)h + h^2}{h} \]

\[ = 3(2^2) + 3(2)(0) + 0^2 = 12 \]


(a) Since

\[ \lim_{x \to 0} (-|x|) = \lim_{x \to 0} |x| = 0, \]

we have

\[ \lim_{x \to 0} x^2 \sin(x^2) = 0. \]

(b) Squeeze theorem.

4. Sol.
(a) Compute
\[ f(-1) = 2(-1)^3 - 6(-1) - 2 = 2, \]
\[ f(1) = 2(1)^3 - 6(1) - 2 = -6. \]
Since \( f(-1) > 0 > f(1) \) and \( f(x) \) is continuous on \([-1, 1]\), there exists at least one number \( c \) in the open interval \((-1, 1)\) such that \( f(c) = 0 \).

(b) The intermediate value theorem.

5. Sol.
\[
\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x + 3)(x - 3)}{x - 3} = \lim_{x \to 3} (x + 3) = 3 + 3 = 6.
\]

(a) \[
\lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} (x + 1) = -1 + 1 = 0.
\]
\[
\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} \sqrt{1 - x^2} = \sqrt{1 - (-1)^2} = 0.
\]
(b) \[
\lim_{x \to -1} g(x) = 0.
\]
(c) Compute
\[
\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} \sqrt{1 - x^2} = 0,
\]
\[
\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} x = 1.
\]
Since
\[
\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x),
\]
\[
\lim_{x \to 1} g(x)
\]
does not exist.
(d) Since 
\[ \lim_{x \to -1} g(x) = 0 = g(-1), \]
g is continuous at \( x = -1 \).

(e) Since \( \lim_{x \to 1} g(x) \) does not exist, \( g \) is not continuous at \( x = 1 \).

7. Sol. Before answering the questions, we first simplify the function:
\[ f(x) = \frac{x + 2}{x^2 - 4} = \frac{x + 2}{(x - 2)(x + 2)} = \frac{1}{x - 2}. \]

(a) \[ \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1}{x - 2} = 0. \]

(b) \[ \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1}{x - 2} = 0. \]

(c) A horizontal asymptote is \( y = 0 \).

(d) A vertical asymptote is \( x = 2 \).

(e) \[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{1}{x - 2} = -\infty \]
\[ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{1}{x - 2} = \infty \]

8. Sol.
(a) \[ m = \frac{f(2.7) - f(2)}{2.7 - 2} = 3.3 \]

(b) \[ f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \approx 4.0 \]
9. Sol. The equation of the tangent line to the graph of $f$ at $x = 2$ is

$$y - f(2) = f'(2)(x - 2) \Rightarrow y - 13 = 20(x - 2) \Rightarrow y = 20x - 27$$