HW2

(Due Tuesday, September 24, 2013)

1. Prove that every positive integer has only finitely many divisors. You may use the fact that \( \mathbb{N} \) is well ordered.

2. In each case find the quotient and remainder.
   (a) \( 19 = (\underline{\text{}}) \times 7 + (\underline{\text{}}) \)
   (b) \( -19 = (\underline{\text{}}) \times 7 + (\underline{\text{}}) \)
   (c) \( 7 = (\underline{\text{}}) \times 19 + (\underline{\text{}}) \)
   (d) \( 10^{19} = (\underline{\text{}}) \times 10^7 + (\underline{\text{}}) \)
   (e) \( 10^{19} - 1 = (\underline{\text{}}) \times (10^7 - 1) + (\underline{\text{}}) \)

3. Use the extended Euclidean algorithm to find an integer solution of

\[ 12345x \equiv 123456 \mod 1234567. \]

4. Find all integer solutions of \( 123x \equiv 456 \mod 789. \)

5. Answer TRUE or FALSE to each of the following statements.
   (a) \( 3 \mid 6 \) \hspace{1cm} (True/False)
   (b) \( 6 \mid 3 \) \hspace{1cm} (True/False)
   (c) Every positive integer is divisible by 1. \hspace{1cm} (True/False)
   (d) Every positive integer is divisible by 0. \hspace{1cm} (True/False)
   (e) Every positive integer is divisible by \(-1\). \hspace{1cm} (True/False)
   (f) The integer 1 is divisible by every positive integer. \hspace{1cm} (True/False)
   (g) The integer 0 is divisible by every positive integer. \hspace{1cm} (True/False)
   (h) The integer \(-1\) is divisible by every positive integer. \hspace{1cm} (True/False)
   (i) Every integer is divisible by itself. \hspace{1cm} (True/False)
   (j) Every integer has at least two divisors. \hspace{1cm} (True/False)